One approach to Monte Carlo simulation of equity options

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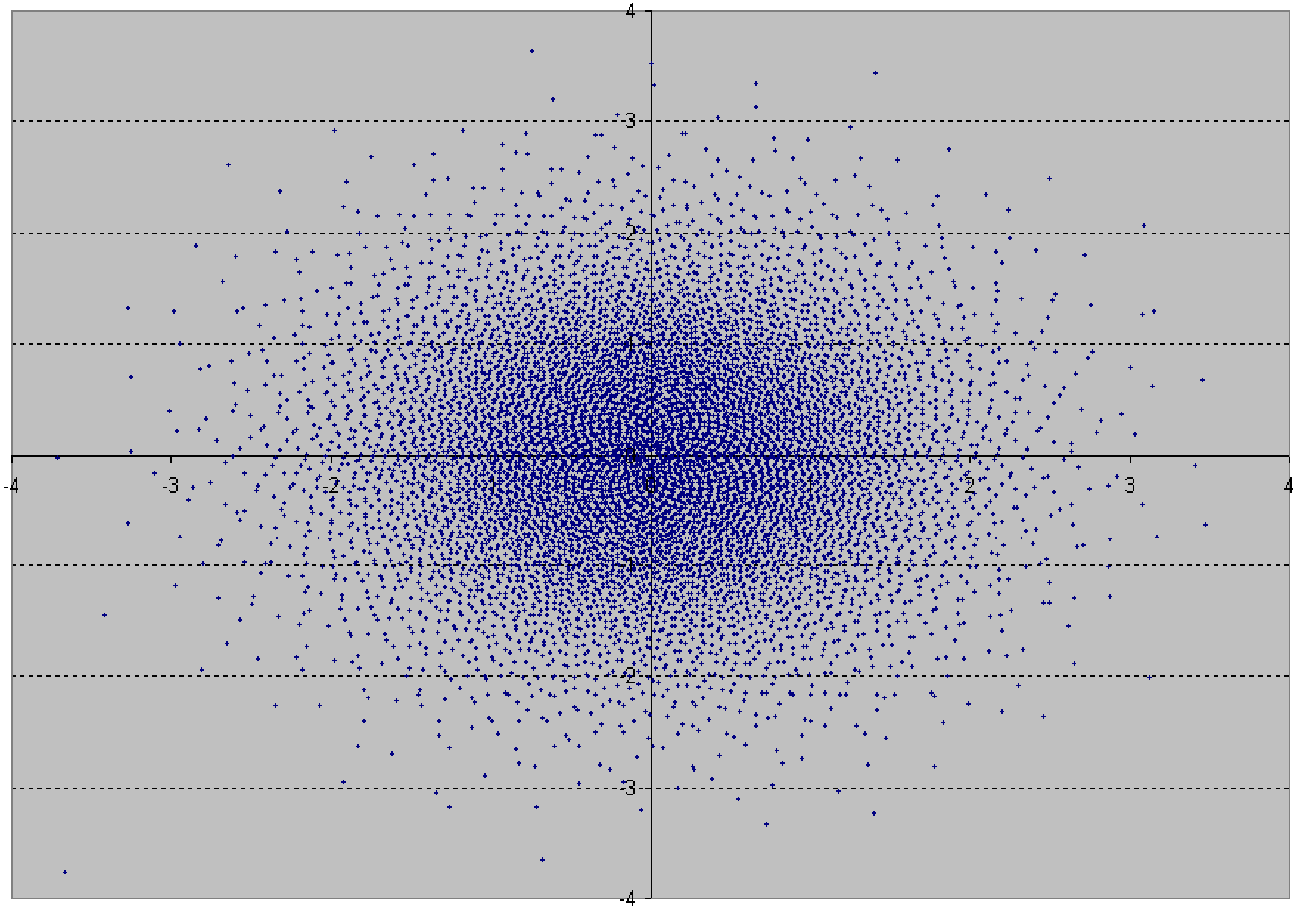
Overview

- Monte Carlo vs Scenarios
- Multidimensional jump–diffusion model
- Non-synchronous closing prices
- ADRs, forwards, futures
- Illiquid stocks
- Large portfolios (many options)
- Liquidity adjusted VaR / Expected Shortfall

Monte Carlo vs Scenarios

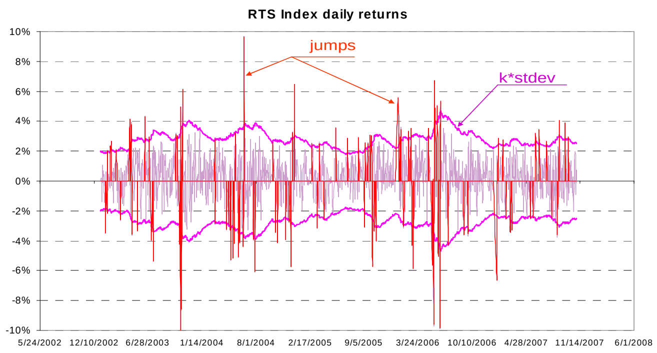
Monte Carlo	Scenarios (grid)
Option Clearing Corporation: STANS ← • System for Theoretical Analysis and Numerical Simulations (Aug. 2006) • Expected Shortfall	Typically used by exchanges • TIMS • SPAN and SPAN-type (Eurex) • FORTS system • PCA (some banks)
IT progress Algorithmic acceleration	Fast, could be used on-line for DMA (NSE of India, FORTS, MICEX)
Flexible Methodologically uniform Easier to develop software	“Produces intuitively more clear results” But: • intra-, intercommodity spreads • volatility moves • large price deviations (fat tails) are treated in a simplified way

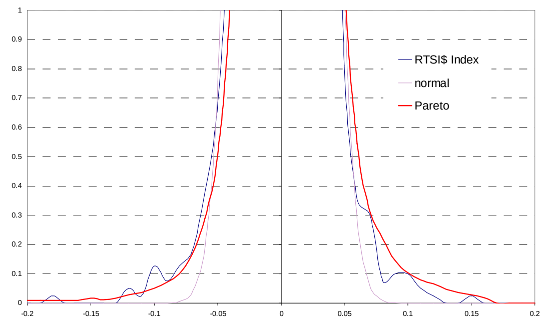
Sobol Sequences



Multidimensional jump–diffusion model

Separation of “regular” price changes and jumps





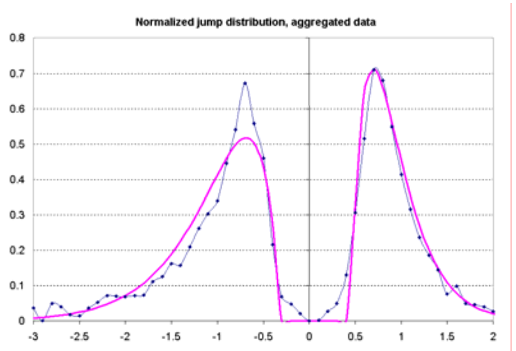
$$\xi_n = \ln(\frac{S_n}{S_{n-1}})$$

$$\alpha = 0.03 \qquad k = 2$$

$$\sigma_n^2 = (1-\alpha)\sigma_{n-1}^2 + \alpha \min(\xi_n^2, k^2\sigma_{n-1}^2)$$

Amplitudes

$$Jump_{stock\_i} = \beta_{stock\_i}^{jump}\Theta + \varepsilon_{stock\_i}$$



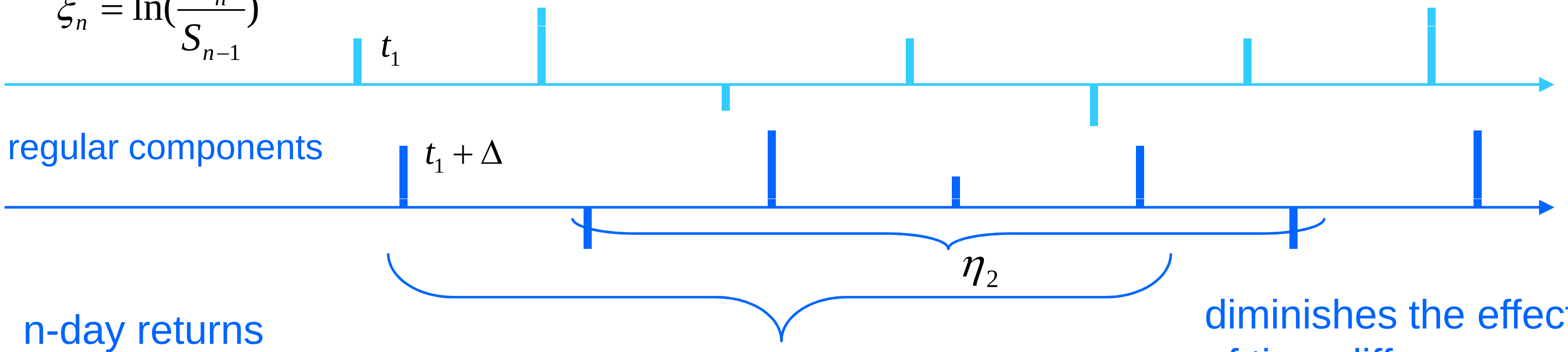
Pareto tails

Occurrences

Jump combinations									Probability
1	0	1	1	0	0	0	0	0	0.020
0	1	1	0	0	1	0	0	1	0.036
1	1	0	1	0	0	0	1	0	0.028
0	0	0	0	1	0	0	0	0	0.064
0	0	1	1	0	0	0	1	1	0.014
…	…	…	…	…	…	…	…	…	…

Non-synchronous closing prices

$$\xi_n = \ln(\frac{S_n}{S_{n-1}})$$

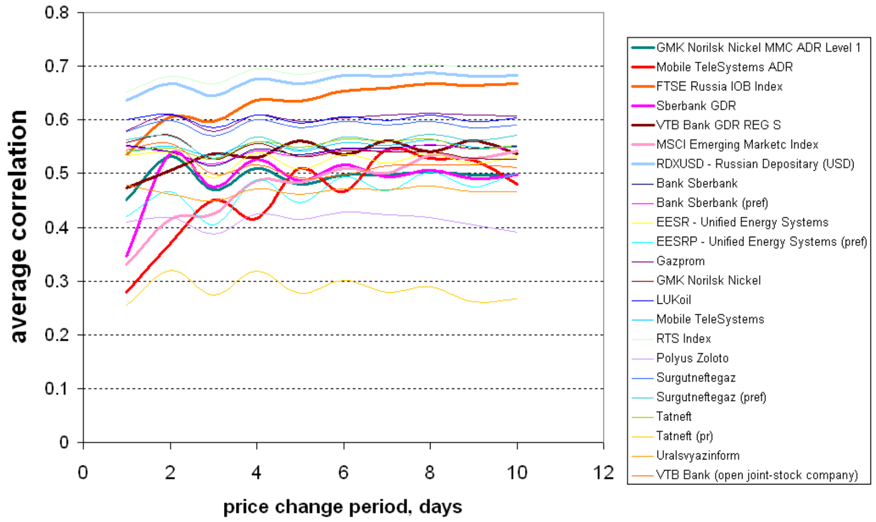


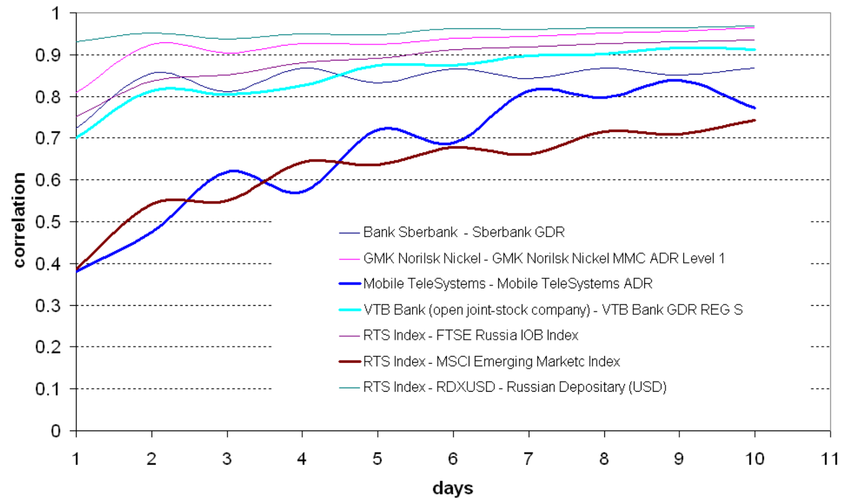
regular components

n-day returns

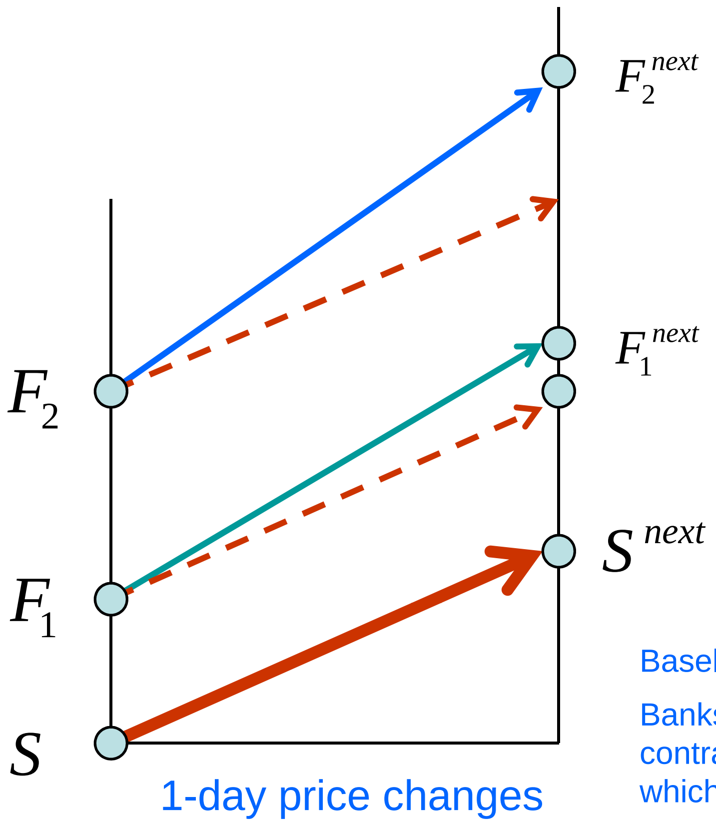
diminishes the effect of time difference

$$\eta_1 = \xi_1 + \xi_2 + \ldots + \xi_5$$





ADRs, Forwards, Futures



$$F_2^{next} = F_2 * \frac{S^{next}}{S} * \exp\left(\Delta r_{common} * t_2 + \Delta r_2 * t_2\right)$$

$$F_1^{next} = F_1 * \frac{S^{next}}{S} * \exp\left(\Delta r_{common} * t_1 + \Delta r_1 * t_1\right)$$

links to underlying

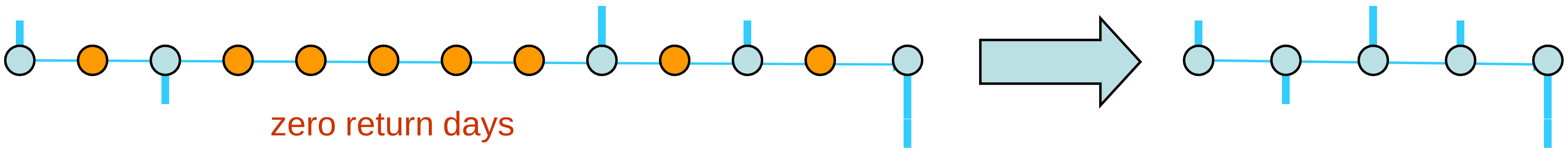
Basel II:

Banks using portfolio strategies including forward and derivative contracts are exposed to variety of additional risks (to directional) which may be well larger than the risk of a change in spot prices:

- Basis risk
- Interest rate risk
- Forward gap risk

Not liquid stocks

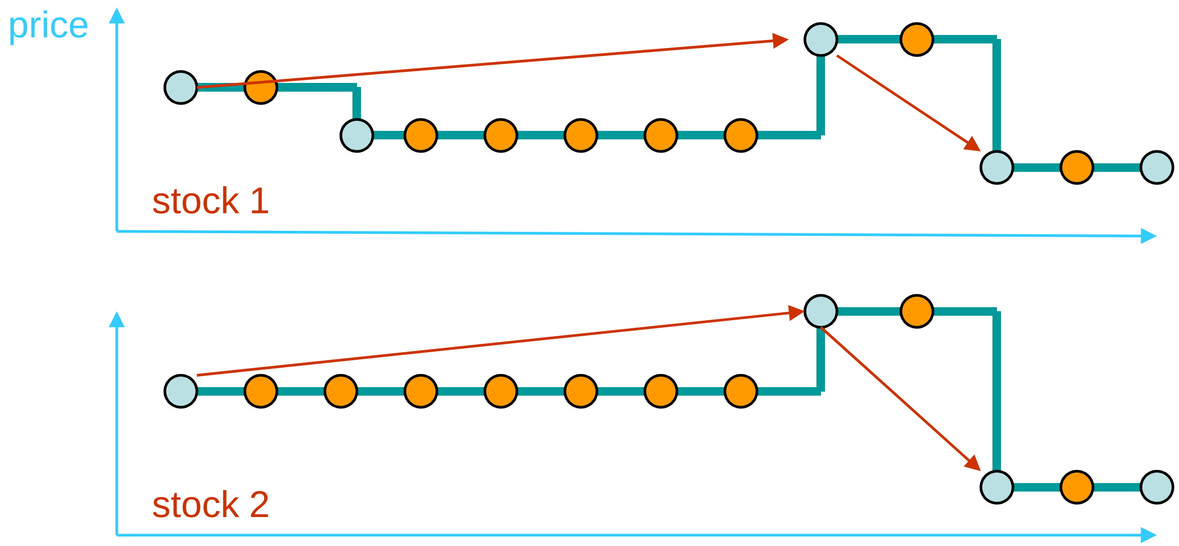
- volatility: excluding zero returns $\xi_n = \ln(\frac{S_n}{S_{n-1}})$



- such a compression implicitly incorporates a liquidity factor adjustment
- cap and floor taken from liquid stocks are applied (median and 95th percentile)

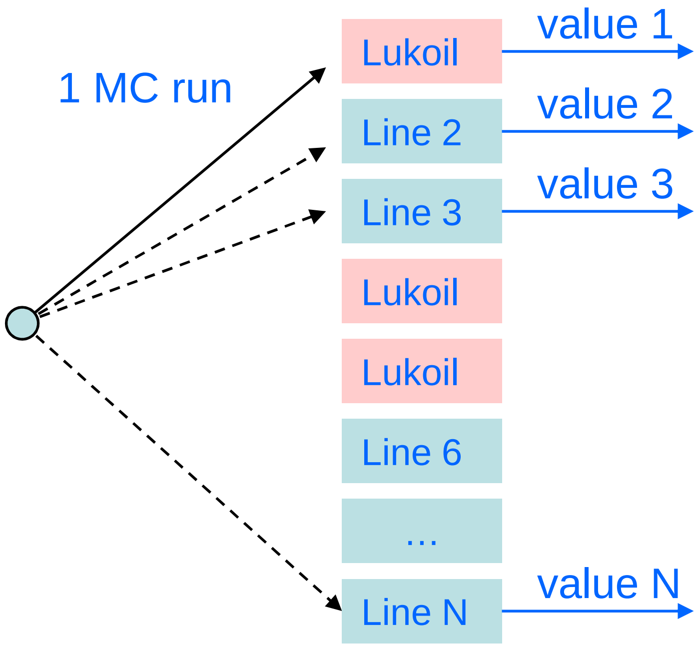
- correlations:

  synchronous non-zero returns only



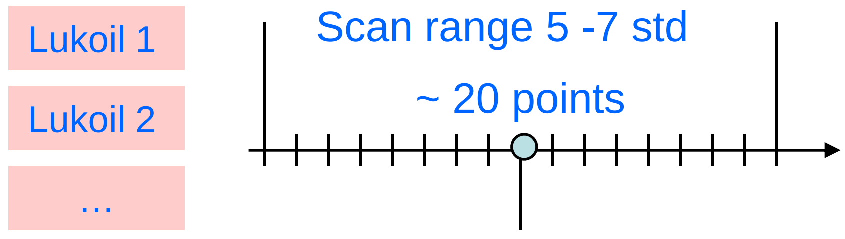
Large portfolios

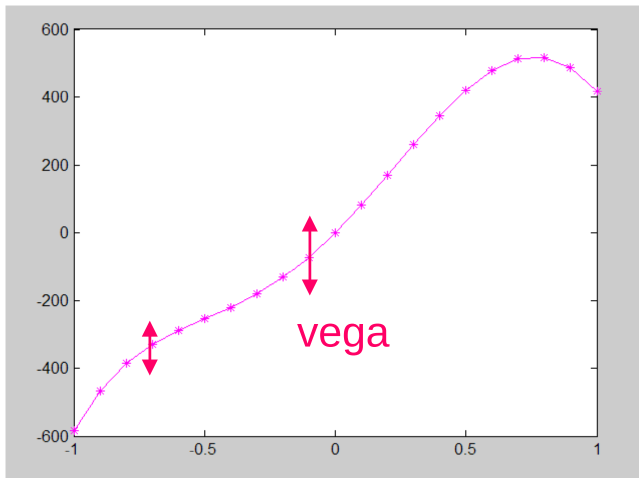
N - total number of stocks, indices, forwards, futures, options in the portfolio



Total = value of portfolio

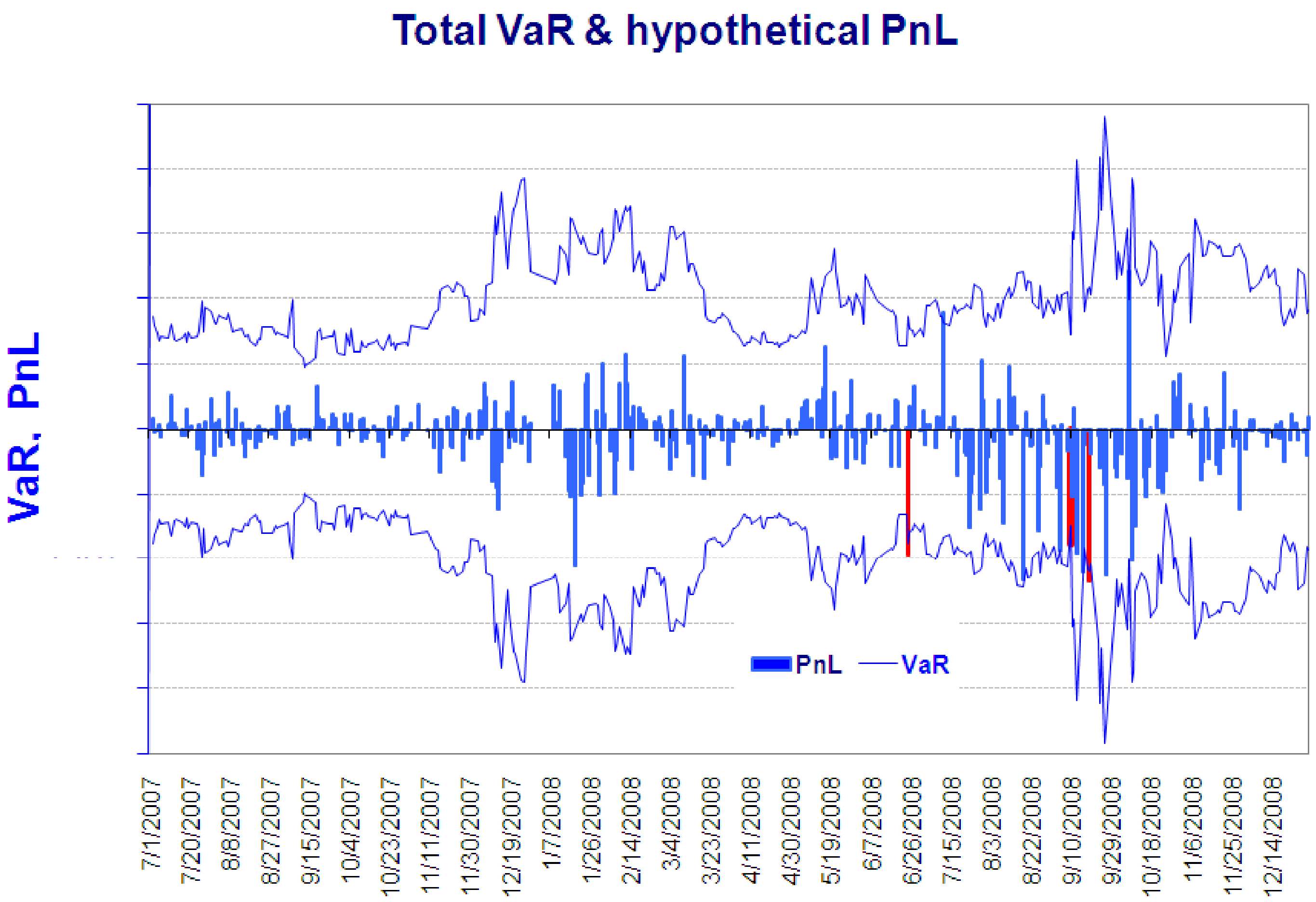
10 000 runs x 10 000 lines





10 000 runs x underlyings

Backtest results for actual trading portfolio



Liquidity Adjusted VaR / Shortfall

$$dS_t = \sigma S_t\, dW_t'$$

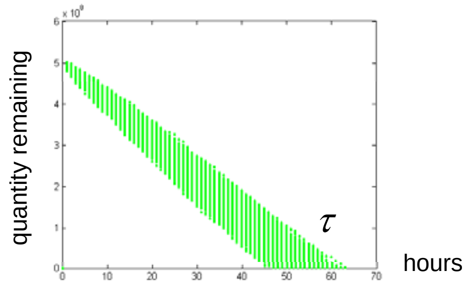
$$dX_t = -L\, dt + \delta\, L dW_t''$$

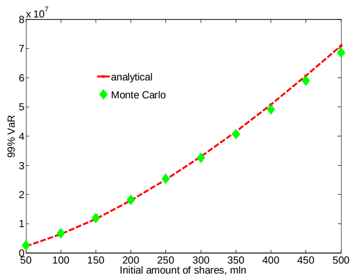
$$dY_t = -S_t\, dX_t = S_t L dt - \delta L S_t\, dW_t''$$

$$Y_\tau \sim N(V, \sigma_{Y_\tau})$$

$$V = S_0 X_0 \qquad \sigma_{Y_\tau} = \frac{1}{\sqrt{3}} V \sigma \sqrt{\tau_0}$$

$$\tau_0 = \frac{X_0}{L}$$





$$R = \begin{pmatrix} 1 & \rho_{12} & \dots & \rho_{1k} \\ \rho_{21} & 1 & \dots & \rho_{2k} \\ \dots & \dots & \dots & \dots \\ \rho_{k1} & \rho_{k2} & \dots & 1 \end{pmatrix}$$ - stock correlation matrix

u_i - average value of i-th stock which can be bought or sold in one day (in USD)

V_i - initial market value of *i-th* stock (negative for short position)

$\tau_i = \frac{V_i}{u_i}$ - time in days needed to close out position

$V = \sum_{i=1}^{n} V_i$ - total initial market value of the position

$$\sigma_{Y_\tau}^2 = \sum_{i,j=1}^{k} \rho_{i,j} \sigma_i \sigma_j u_i u_j (t_{ij}^{\min})^2 \left(\frac{1}{3} t_{ij}^{\min} + \frac{1}{2} \left(t_{ij}^{\max} - t_{ij}^{\min} \right) \right)$$

$$t_{ij}^{\min} = \min(\tau_i, \tau_j) \qquad t_{ij}^{\max} = \max(\tau_i, \tau_j)$$