Zero-coupon yield curves before and after liquidity crisis

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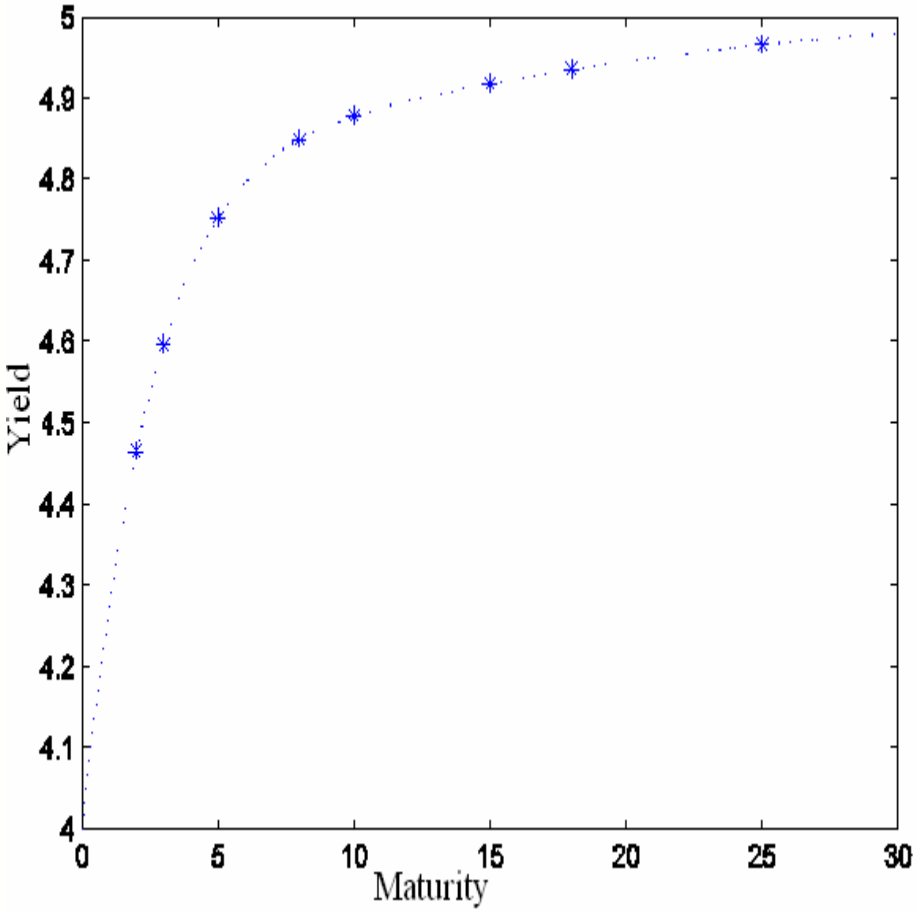
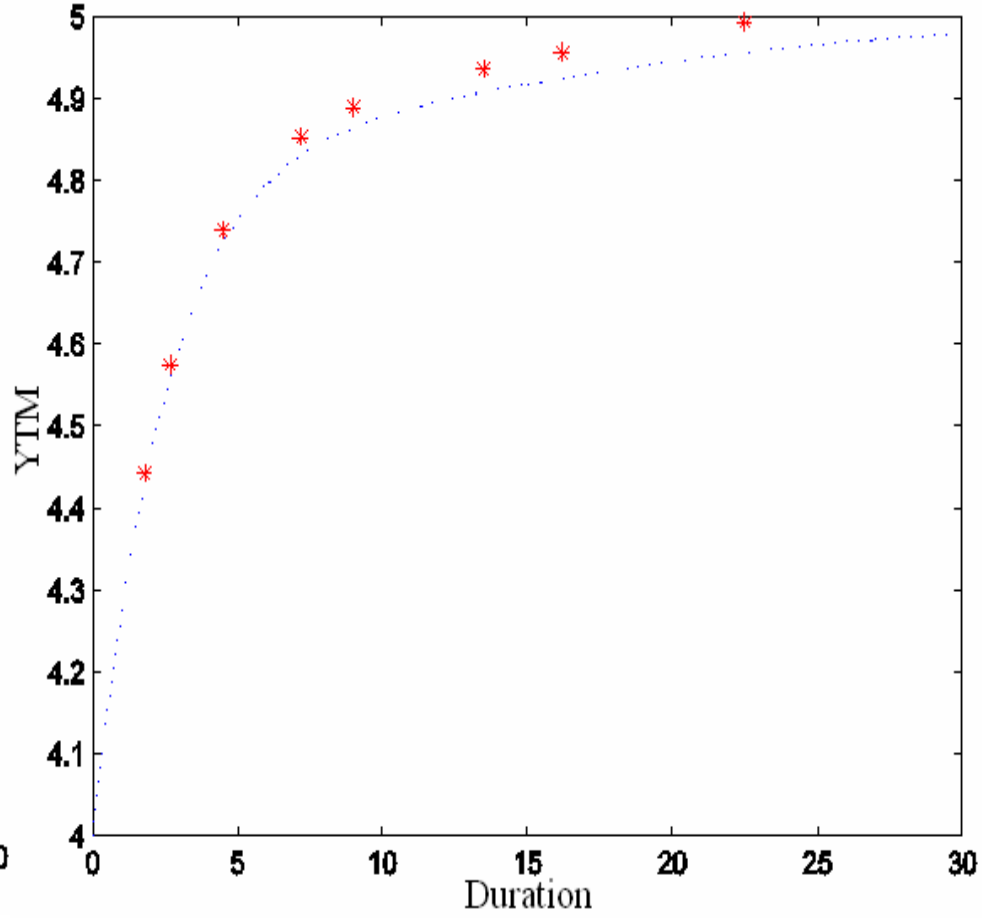
Contents

- Constructing zero-coupon yield curve.
- Data and Market situation.
- G-curve implementation problems: stress-testing results.
- Yield curves July-September 2007.
- Yield curves January-March 2008.

Yield Curve


Yield
Maturity
5
4.9
4.8
4.7
4.6
4.5
4.4
4.3
4.2
4.1
4
0
5
10
15
20
25
30


YTM
Duration
5
4.9
4.8
4.7
4.6
4.5
4.4
4.3
4.2
4.1
4
0
5
10
15
20
25
30

The Problem: Formalization

- Zero-coupon bonds: $P_k = N_k d(t_k)$

$$d(t) = e^{-r(t)t}$$

- Coupon-bearing bonds: $P_k \approx \sum_{i=0}^{n} F_{i,k} d(t_i)$

$$\mathbf{P} \approx \mathbf{F}\mathbf{d} \qquad \mathbf{d}_i = d(t_i)$$

- The system is underdetermined.
- Discount function values are desired in intermediate points.

A Priori Assumptions

- Lack of available data forces one to make additional assumptions.

- Different assumptions lead to different problem statements and therefore to different results.

Different Approaches

- Assumptions on a specific parametric form of the yield curve (parametric methods)

- Assumptions on the smoothness (in some sense) of the yield curve (spline methods).

For detailed presentation on yield curve construction techniques see
http://www.riskconference.ru/presentation/2006/Smirnov.pdf

Problem statement

- Seek the solution in the form:

$$d(t) = \exp\left[-\int_0^t f^2(\tau)\right]d\tau$$

- Smoothness: $\int_0^T f'^2(\tau)d\tau \to \min$

$$\sum_{k=1}^{N} w_k \left(\sum_{i=1}^{n} exp\left[-\int_0^{t_i} f^2(\tau)d\tau\right] F_{i,k} - P_k \right)^2 + \alpha \int_0^T f'^2(\tau)d\tau \to \min_f$$

The Solution

- *f(t)* – spline of form:

$$f(t)=\begin{cases} C_1\exp\{\sqrt{\lambda_k}(t-t_{k-1})\}+C_2\exp\{-\sqrt{\lambda_k}(t-t_{k-1})\}, & \lambda_k>0 \\ C_1\sin(\sqrt{-\lambda_k}(t-t_{k-1}))+C_2\cos(\sqrt{-\lambda_k}(t-t_{k-1})), & \lambda_k<0, \\ C_1(t-t_{k-1})+C_2, & \lambda_k=0 \end{cases}$$

$$t\in[t_{k-1};t_k] \quad f(t_k-0)=f(t_k+0), \quad f'(t_k-0)=f'(t_k+0)$$

$$\lambda_{k+1}-\lambda_k=-\frac{1}{\alpha}\sum_{s=1}^{N}\left(\sum_{i=0}^{n}F_{i,s}d(t_i)-P_s\right)F_{k,s}d(t_k)$$

Insufficient Bonds

- Bonds of approximately equal credit quality and liquidity are required.
- One needs additional assumptions on correlation between yields of similar bonds in different groups (of credit quality and/or liquidity).

Benefits

- This approach enables one to estimate the yield curve on markets with few instruments of different credit quality and liquidity.
- Numerical values of credit spreads of particular issuers may be of some interest themselves.
- This approach was accepted by European Bond Commission as a Standard on Risk-Free Spot Yield Curve and Credit Spreads.

Part 2

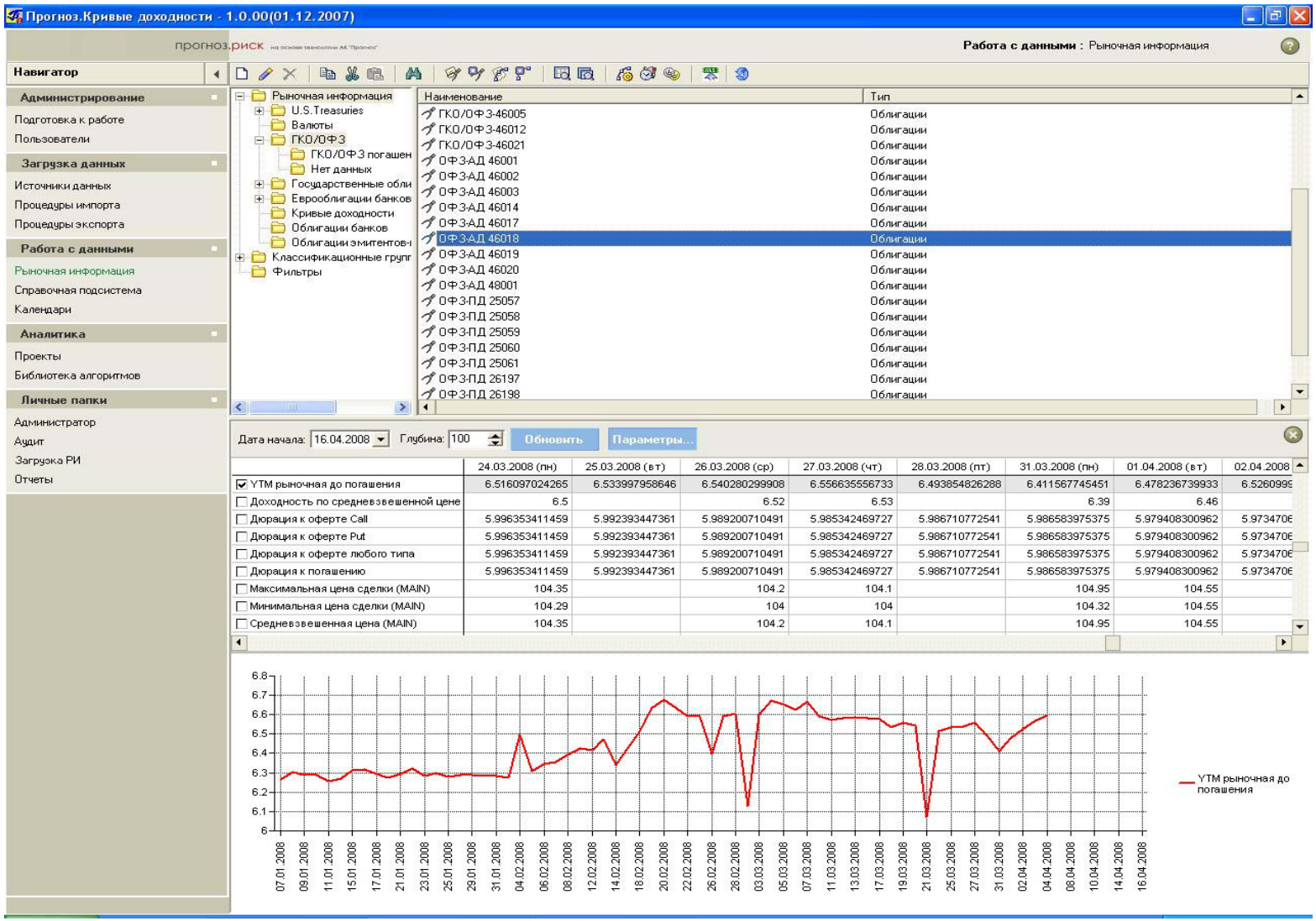
- Constructing zero-coupon yield curve.
- Data and Market situation.
- G-curve implementation problems: stress-testing results.
- Yield curves July-September 2007.
- Yield curves January-March 2008.

Initial Data Preparation

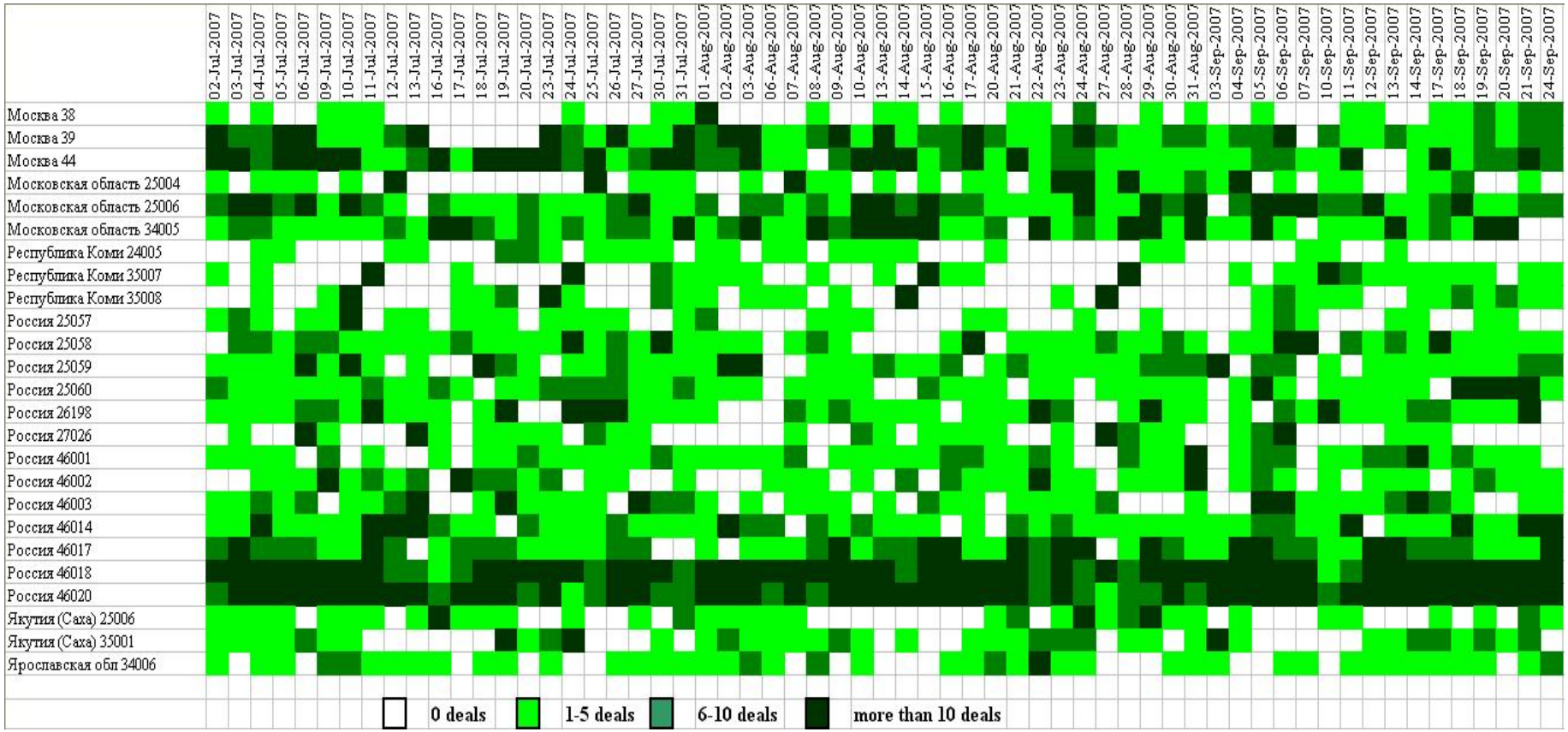
- Data accumulation and storage.
- Filtration.
- Data augmentation.
- Application.

For detailed presentation on Initial Data Preparation see for example
http://www.riskconference.ru/presentation/2007/Smirnov-Kosyanenko.pdf

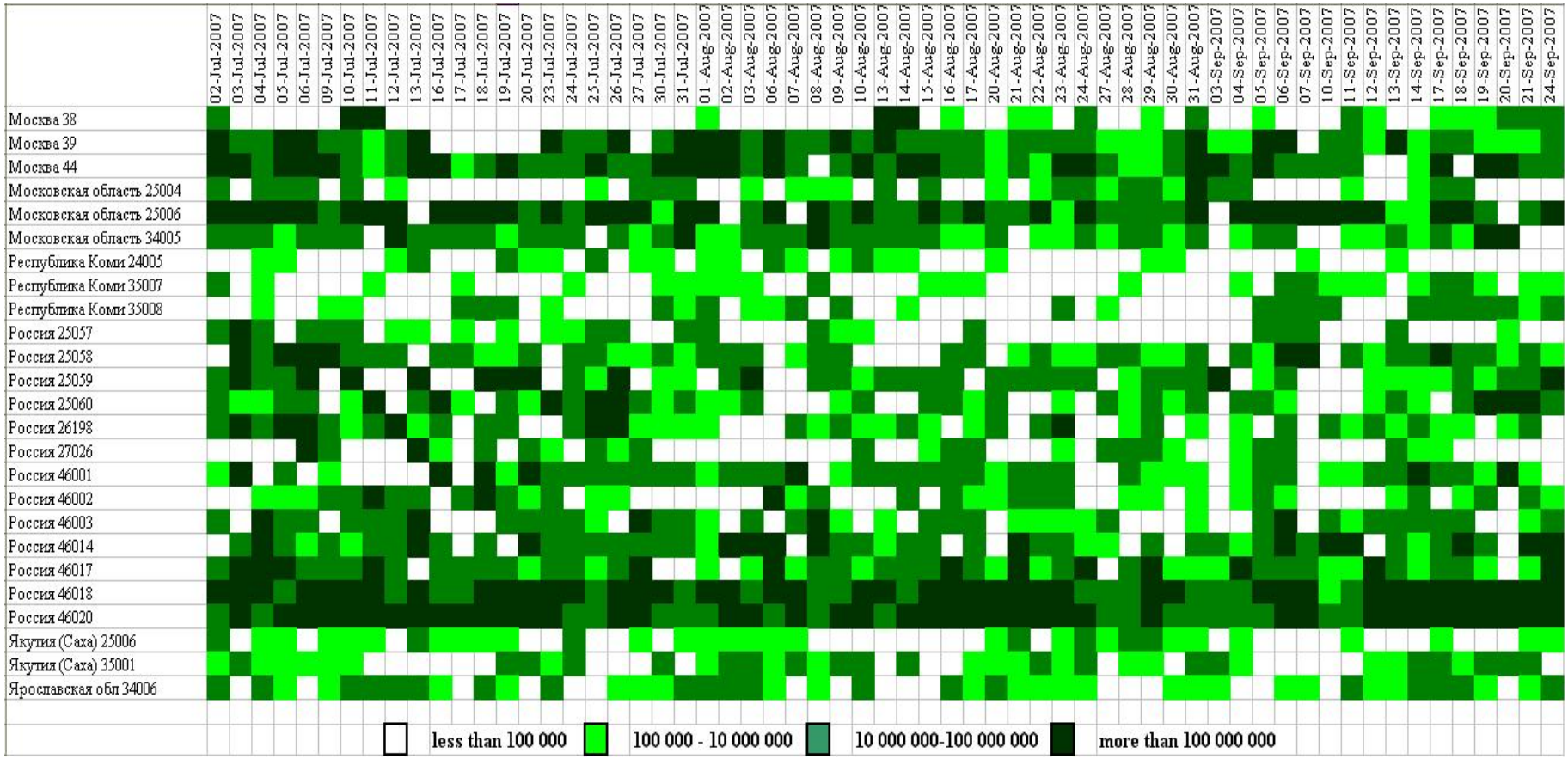
Market data storage



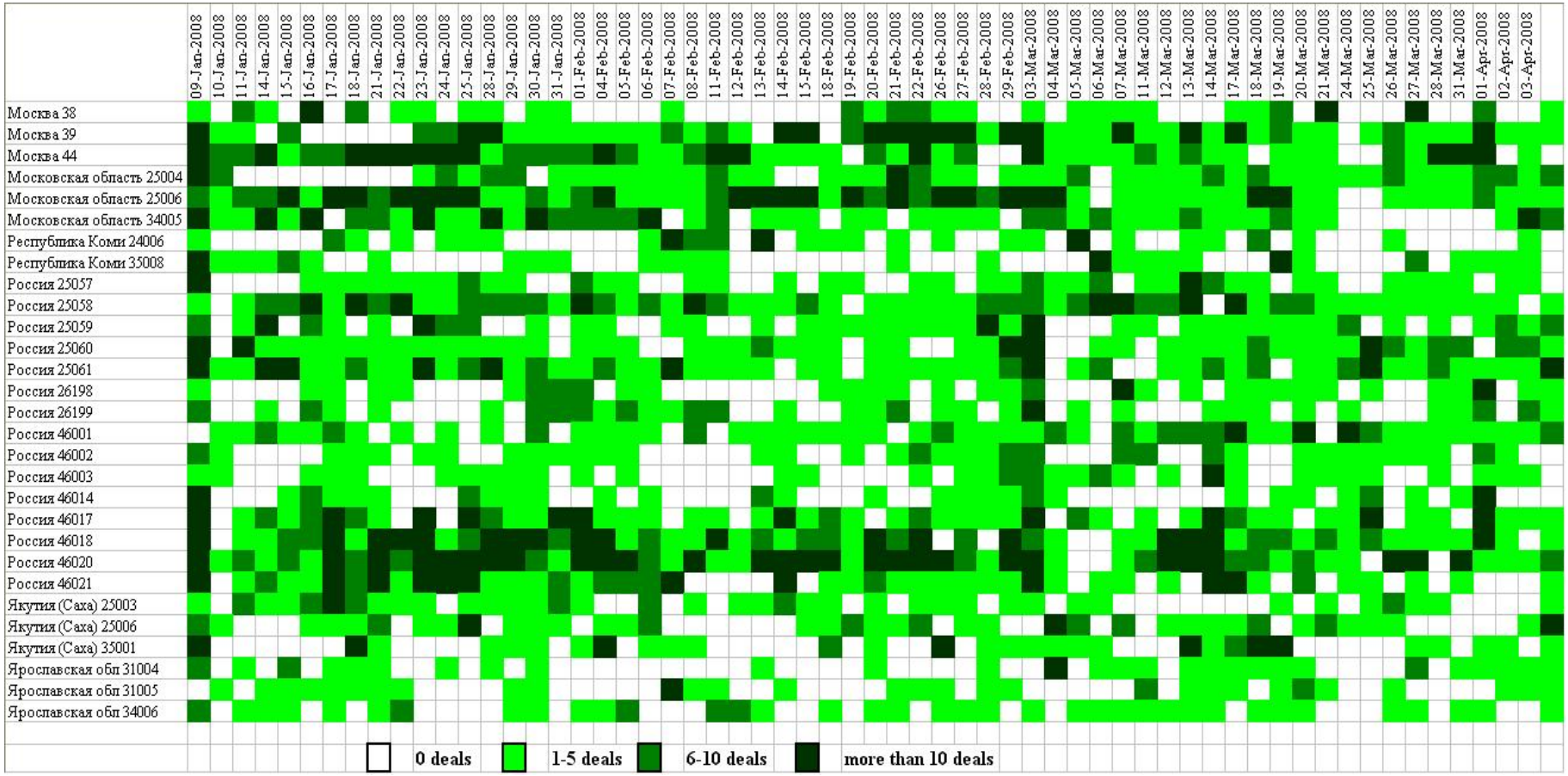
Deals Number July-September 2007



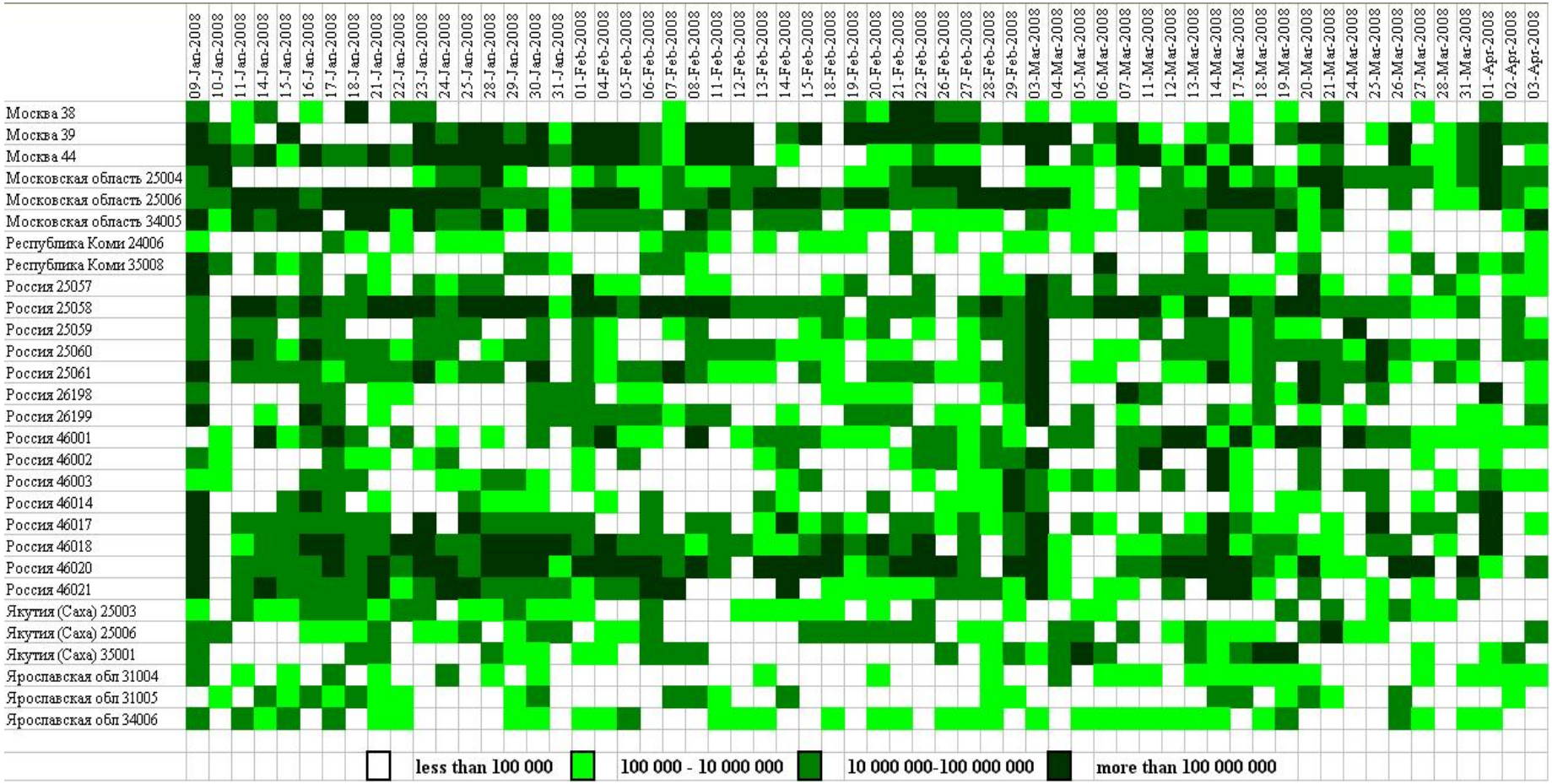
Turnover July-September 2007



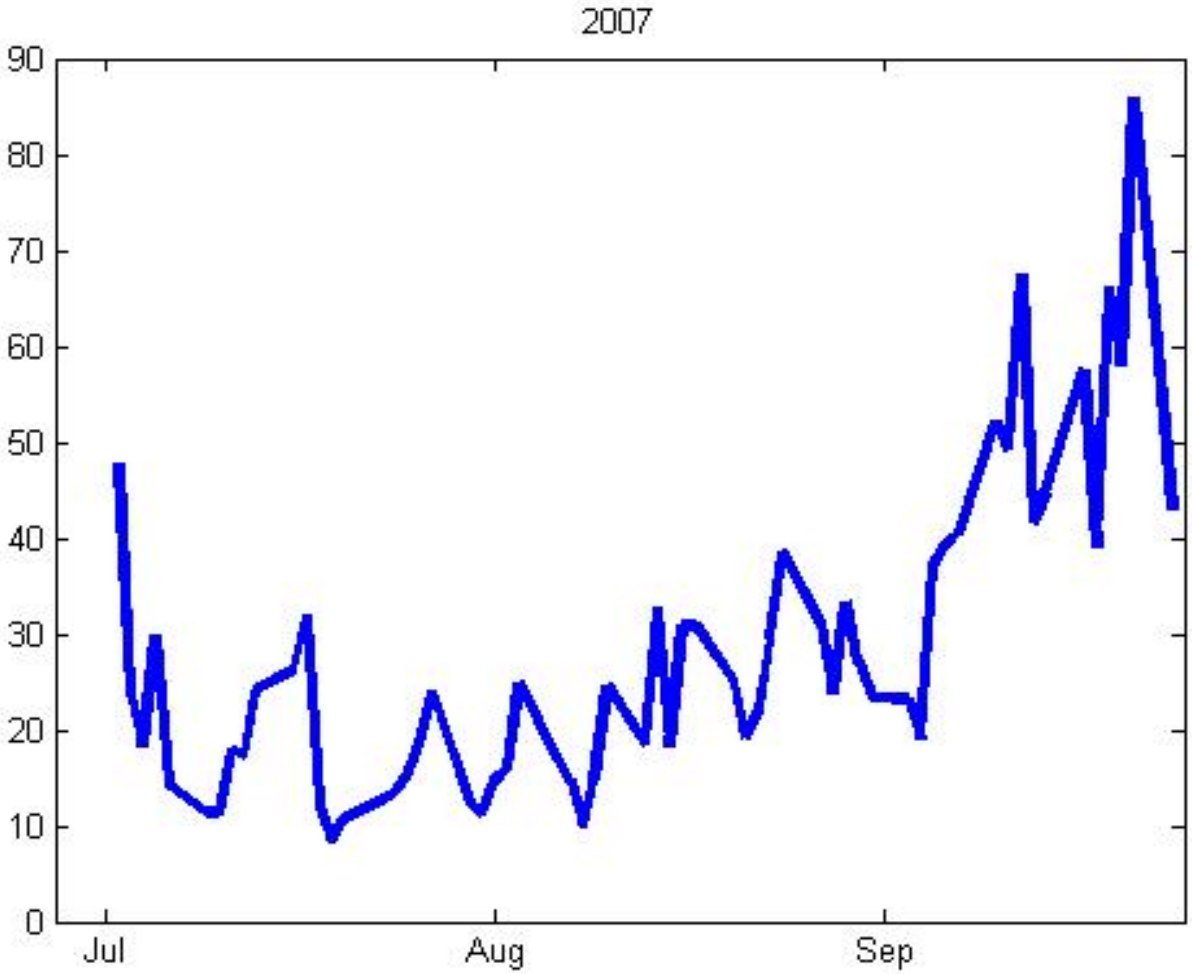
Deals Number January-March 2008



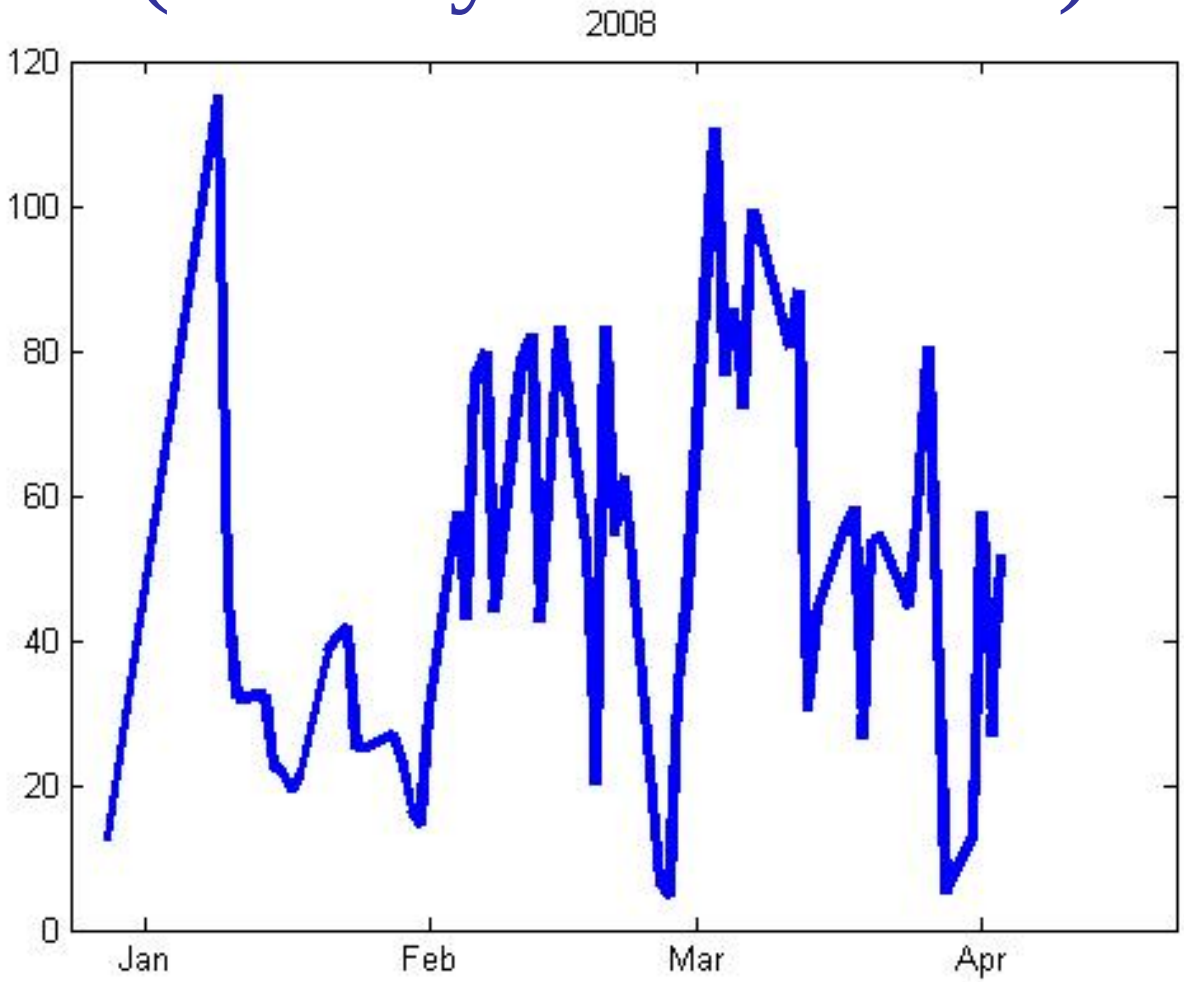
Turnover January-March 2008



Average Bid-Ask Spreads dynamics (July-September 2007)



Average Bid-Ask Spreads dynamics (January-March 2008)



Part 3

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Stress-Testing

- Detailed technique reconstruction.
- Known dynamics of the yield curve.
- The curves belong to the same parametric family as G-curve.
- Fixed bonds base for computation.
- Trades occur in random consequence.

The G-curve model

A Kalman filter analogue (linear) is applied to a nonlinear model:

$$R(t) = \beta_0 + (\beta_1 + \beta_2)\frac{\tau}{t}\left[1 - \exp\left(-\frac{t}{\tau}\right)\right] - \beta_2 \exp\left(-\frac{t}{\tau}\right) +$$

$$+ g_1 \exp\left(-\frac{t^2}{2}\right) + g_2 \exp\left(-\frac{(t-1)^2}{2}\right) + g_3 \exp\left(-\frac{(t-2)^2}{2}\right)$$

The Nelson-Siegel parametrization (4 parameters) with 3 correcting terms with 3 additional parameters; a recurrent parameter estimation algorithm.

Slow adaptation

- Explicitly forced bounds on parameter changing: G-curve moves accordingly to the ratio of the errors. The less accurate the deal is compared to the G-curve accuracy, the more G-curve will respond to this deal.
- This causes the curve to adapt slowly to market changes.
  - This is an advantage since a large outlying point (either random or by mistake) will not change the curve significantly and thus lead to mispricing.
  - On the other hand the curve adapts too slowly to market changes.

Averaging

- Intraday G-curves are averaged by their parameters instead of more reasonable geometric averaging. Due to the non-linearity of the curve with respect to the parameters the averaged curve might not reflect the character of the intraday curves.

Dependence on the order of the deals

- The G-curve methodology is discrete in time while applied to continuous model.
- It does not take into account elapsed time intervals. The resulting curve will be the same, no matter whether several ticks or several days have passed since the previous deal.
- The resulting curve may depend on the exact order of committing two successive deals even if the deals are committed in the adjacent time ticks.

Convergence problems

- The G-curve methodology does converge only asymptotically.
- To converge asymptotically the methodology requires deals to be random and uniformly distributed along the curve (with respect to duration).
- In reality convergence should not be expected.

Parallel shifts

- G-curve tends to respond to every noticeable yield change with a parallel shift. Since the curve reacts slowly to the market changes this shift is broken into several lesser ones.
- This may cause two deals to cancel each other if one of them has a high yield and the other has a low yield. The duration of the deals does not matter since the curve tends to respond with a parallel shift regardless of the duration.

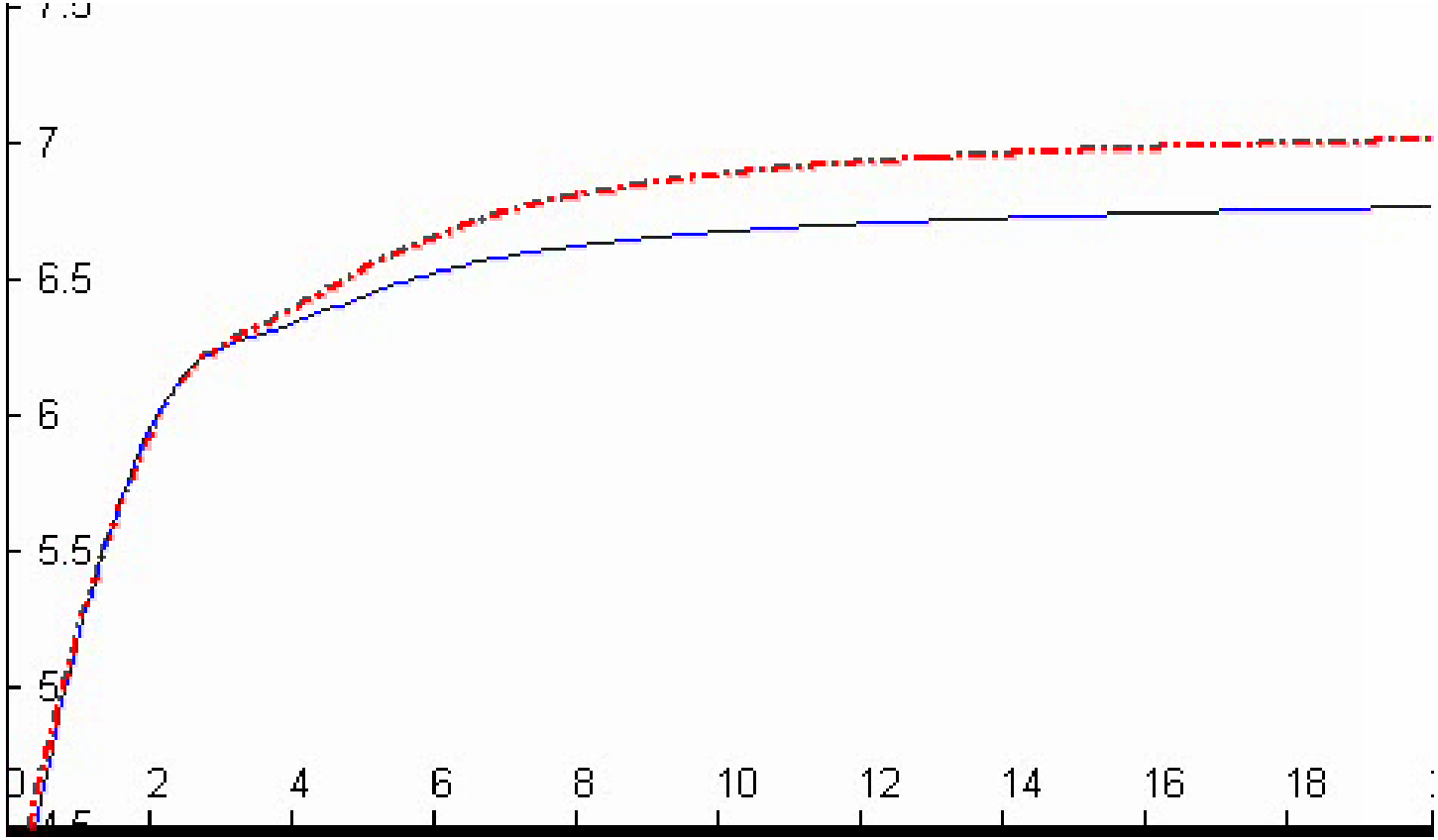
Arbitrage opportunities

- The stochastic dynamics of the interest rates assumed by the G-curve methodology may be considered within the Heath-Jarrow-Morton framework.
- But the so-called HJM drift condition does not hold, which means the presence of arbitrage opportunities.

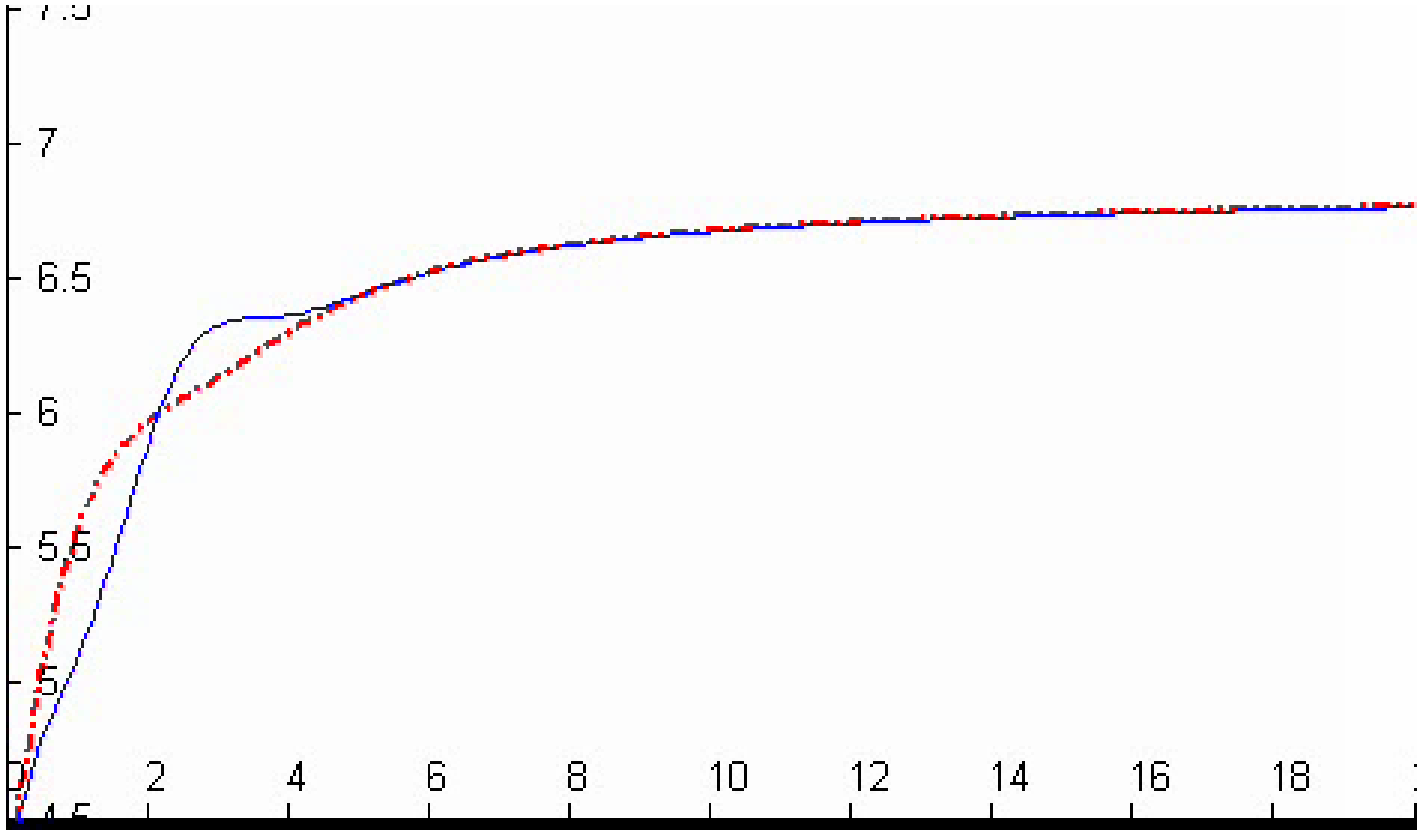
Spot forward rate curve

- Natural assumption is that forward interest rates implied by a yield curve are nonnegative.
- G-curve methodology does not take into account forward interest rates.
- Forward interest rates curve generated by G-curve is nonsmooth and forward rates may be negative.
- Unacceptable for interest rate derivatives pricing.

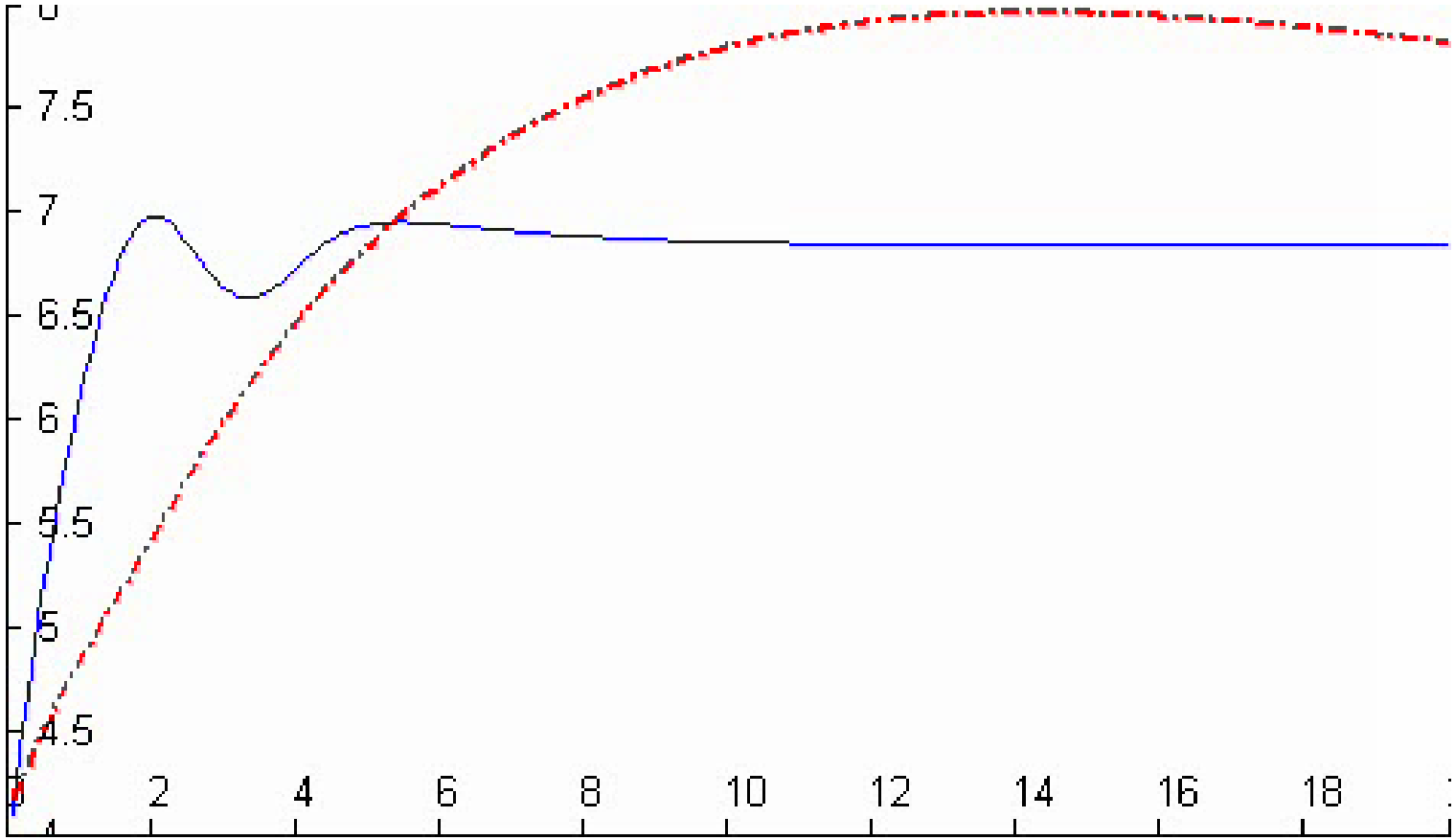
Parallel shifts example



Non-identifiability of the curve shape



Spot forward rate curve



Part 4

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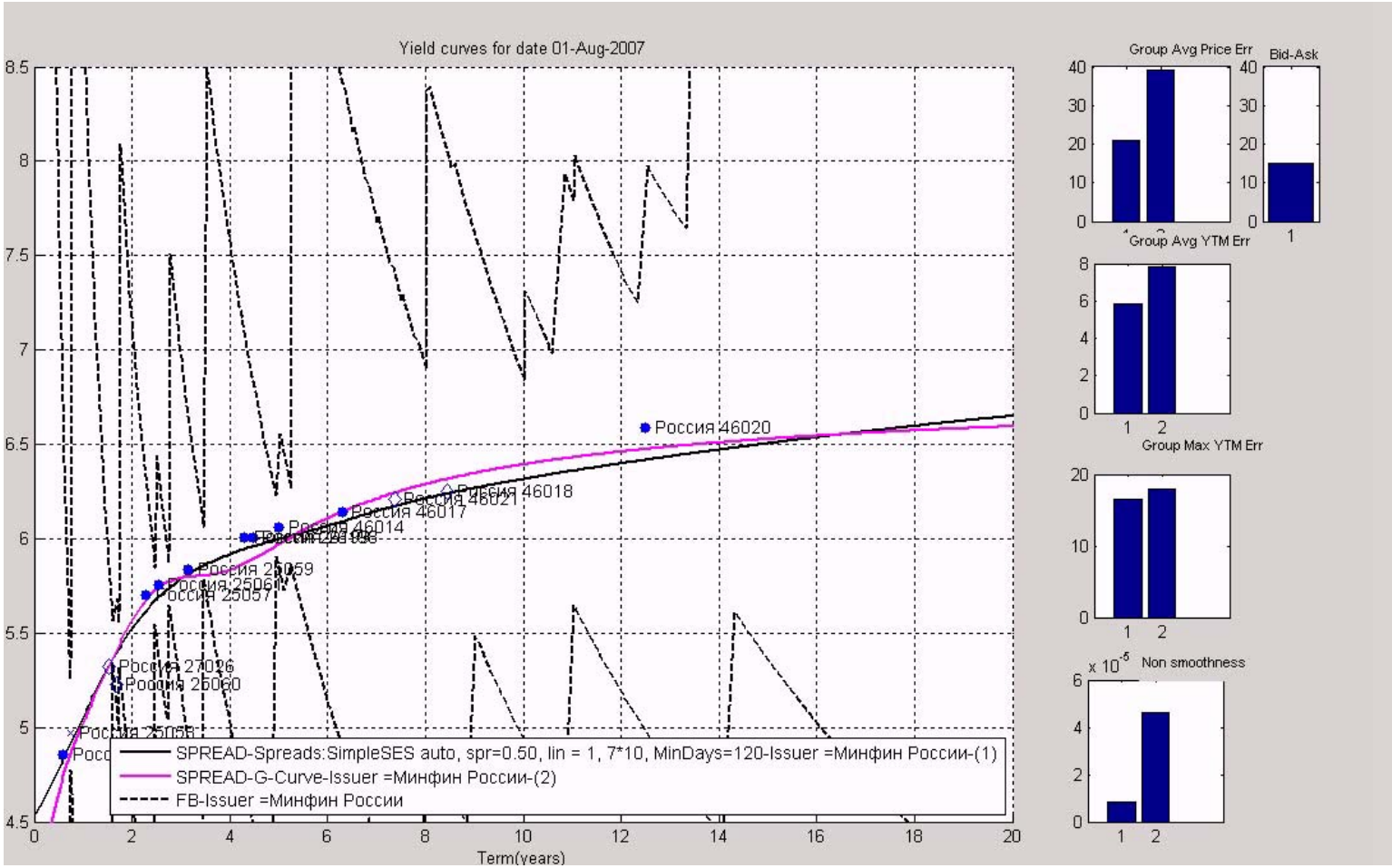
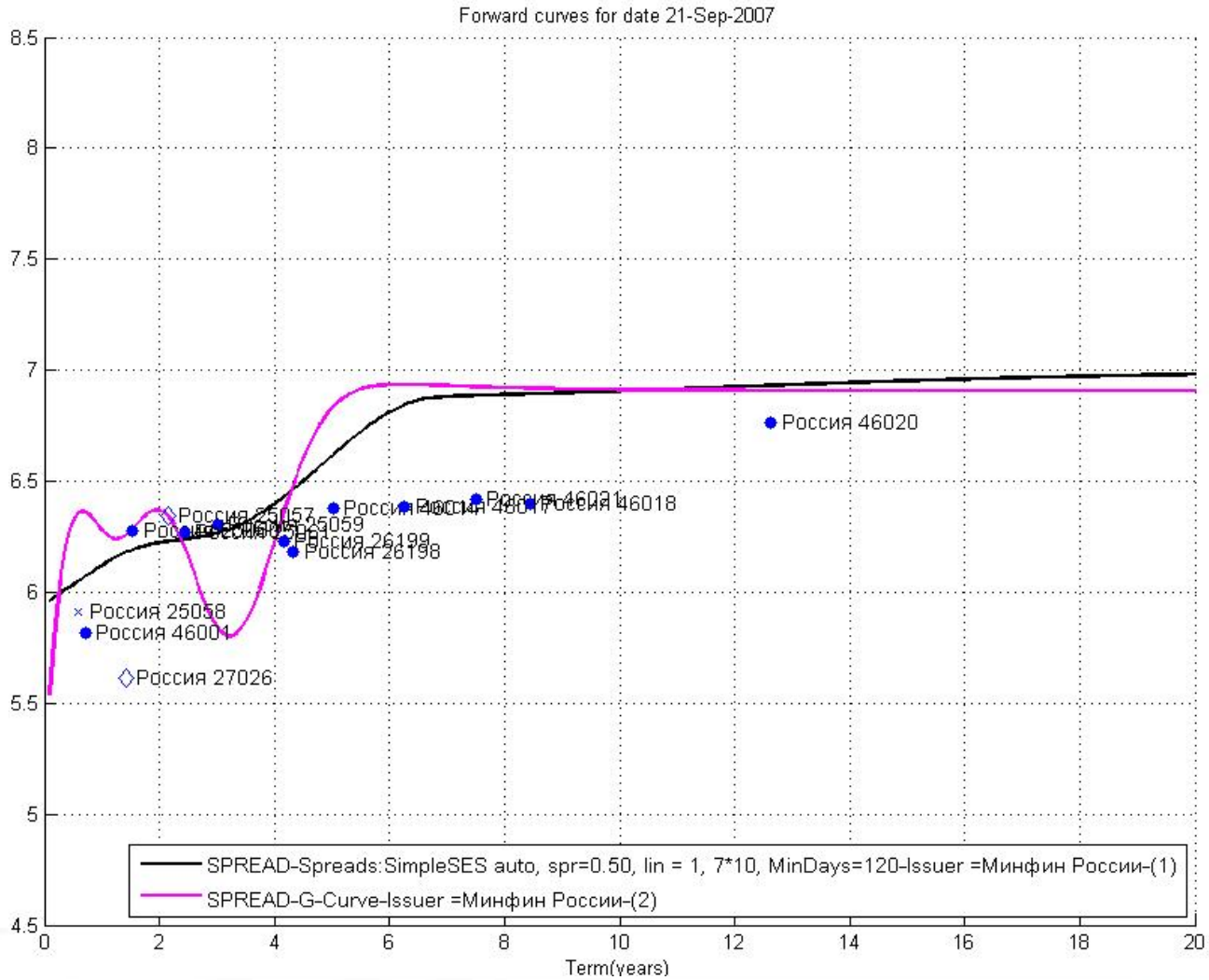
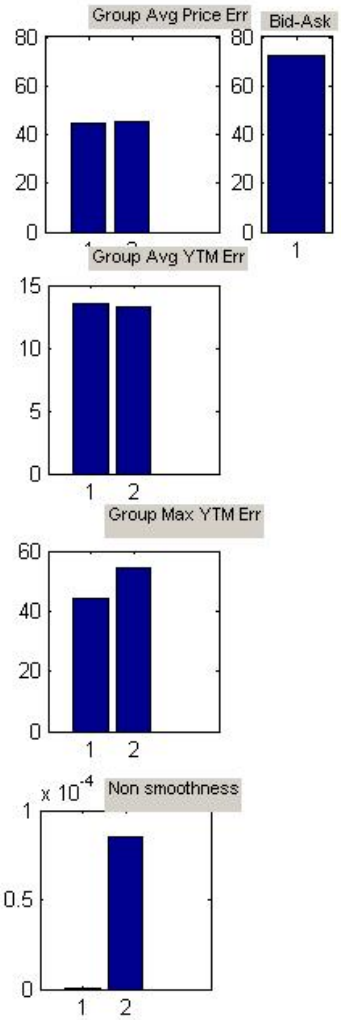
Yield curves dynamics

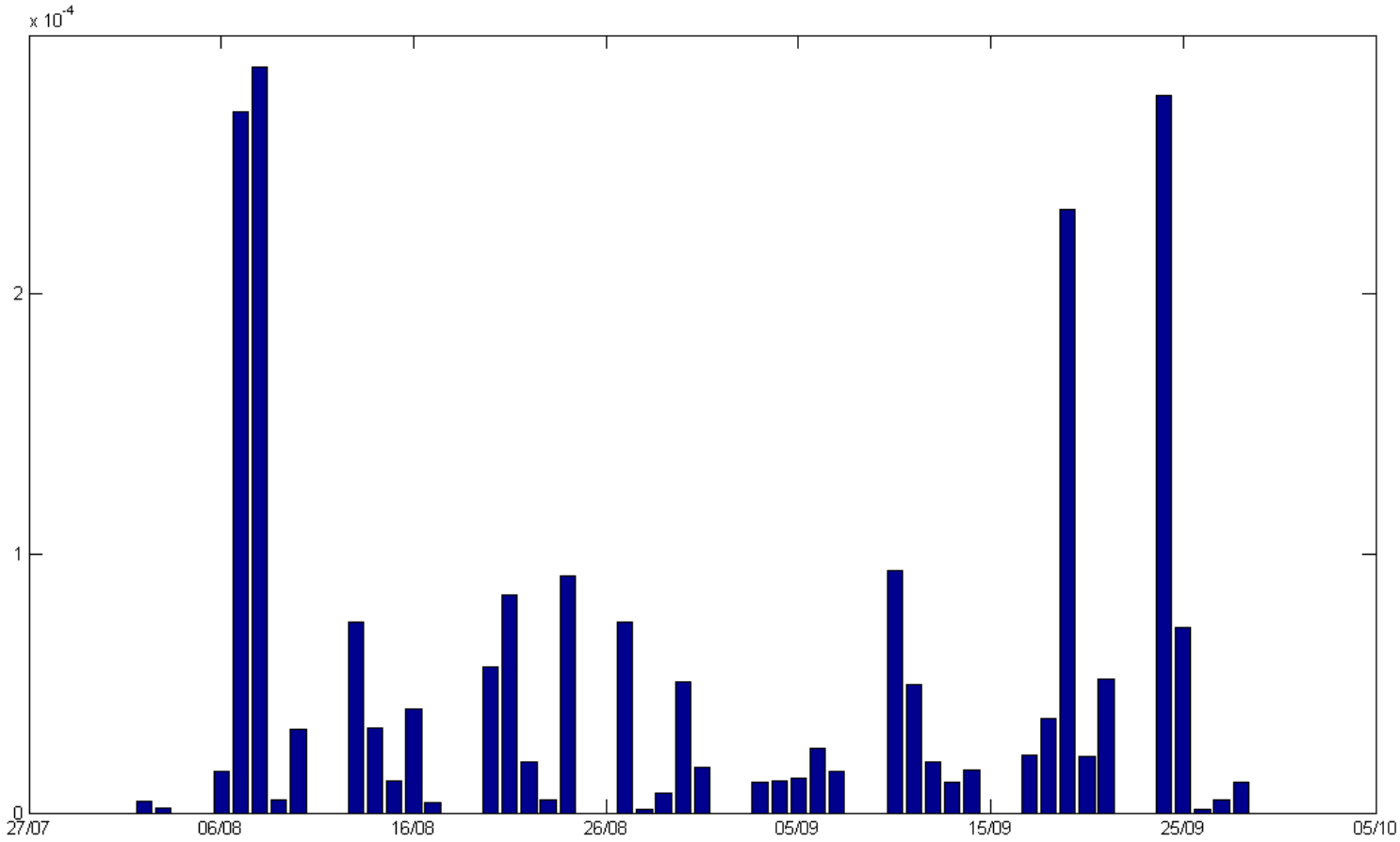


Forward rate curves dynamics





Distance from the previous curve



Part 5

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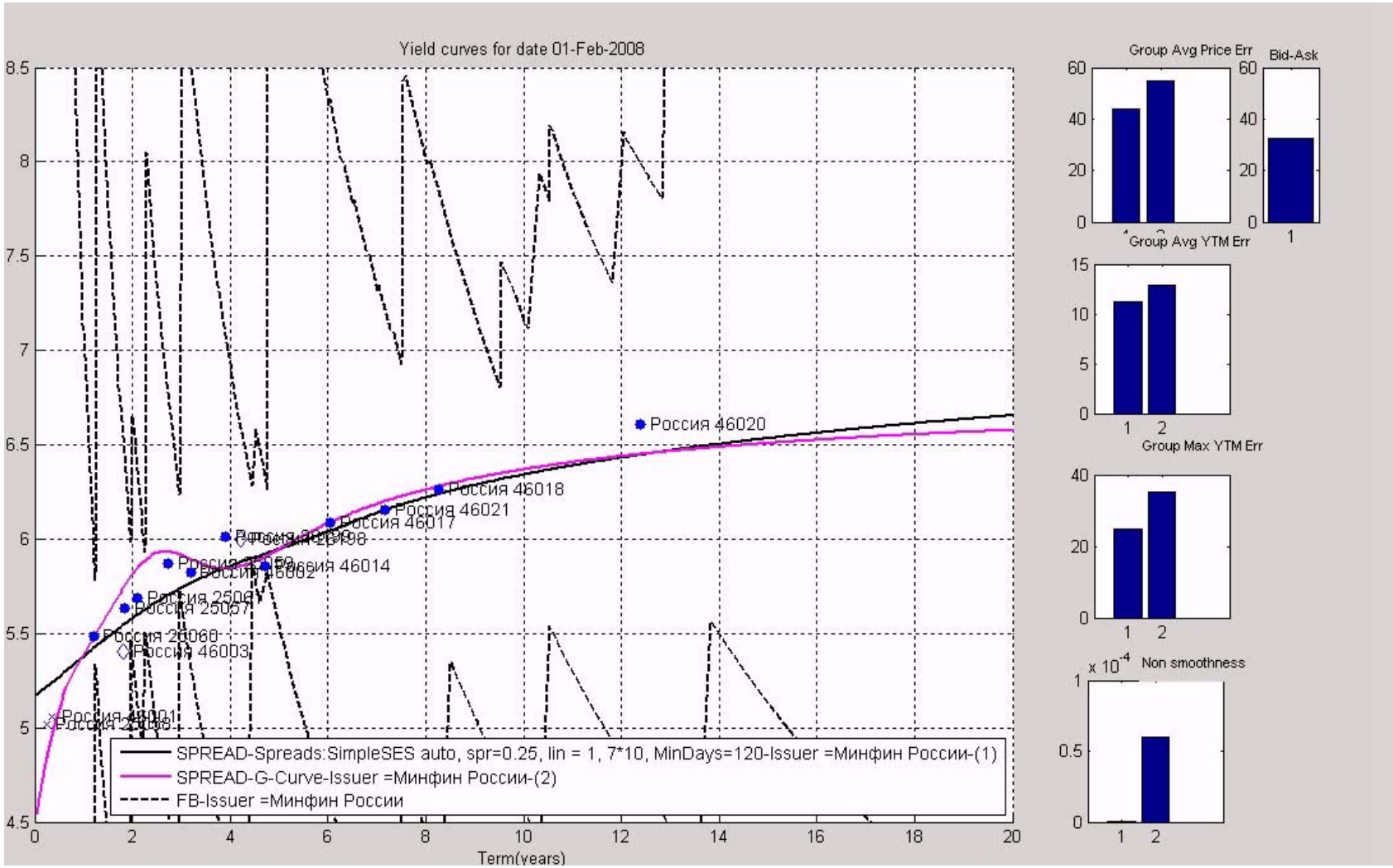
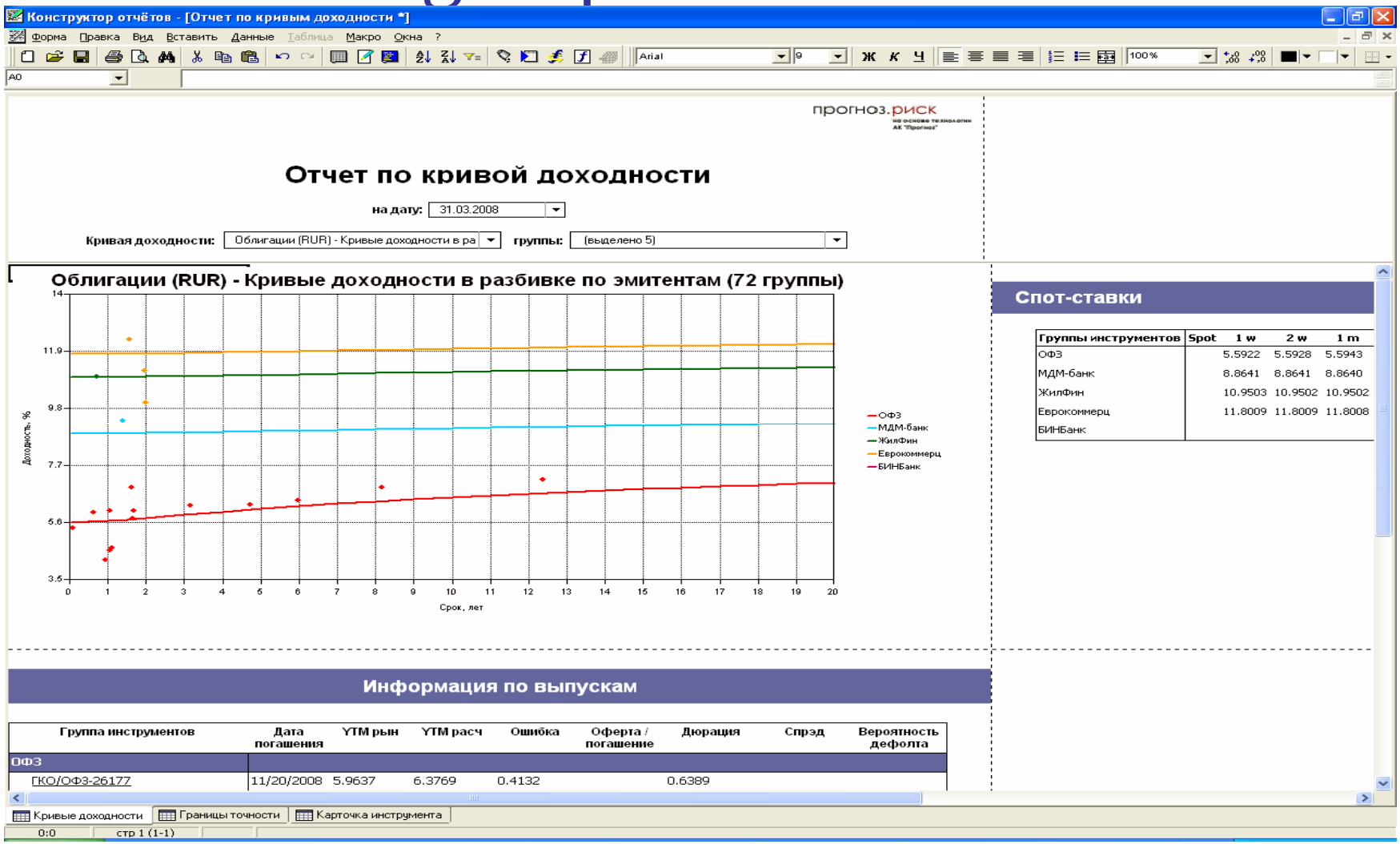
Yield curves dynamics



Yield curves dynamics for several groups of bonds



Market implied default probability

Конструктор отчётов - [Динамика вероятностей дефолта *]

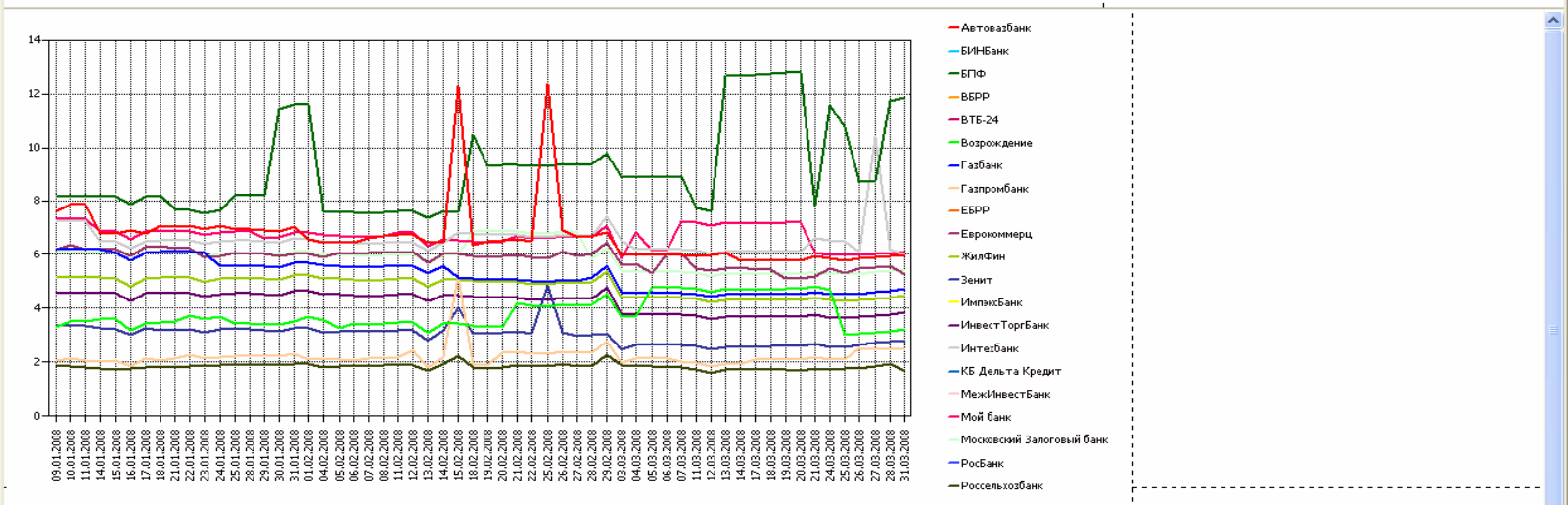
Форма Правка Вид Вставить Данные Таблица Макро Окна ?

прогноз.риск

Динамика вероятностей дефолта

Кривая доходности: Облигации (RUR) - Кривые доходности в ра: **Банки:** (выделено 21)

Период: (выделено 122)



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Автоваз банк	231	7.5897	7.8603	7.8500	6.7672	6.7604	6.9097	6.7863	7.0507	7.0415	7.0493	6.9261	7.0720	6.9340	6.9320	6.9138	6.8646
БИНБанк	2562																
БПФ	1677	8.1410	8.1515	8.1412	8.1503	8.1533	7.8723	8.1573	8.1870	7.6698	7.6677	7.5363	7.6475	8.2112	8.2167	8.2099	11.4424
ВБРР	3287																
ВТБ-24	1623																
Возрождение	1439	3.3204	3.5046	3.4927	3.5924	3.5862	3.1601	3.4204	3.4495	3.4934	3.7158	3.5871	3.6783	3.4134	3.4109	3.3923	3.3683

Динамика вероятностей дефолта

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Thank You for attention!

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