



# How to build yield curves for interest rate swaps and other derivatives

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**Ilya Polimatidi**

**UniCredit Bank / Russia**

**Olga Karnauhova**

**Laboratory for Financial Engineering and Risk  
Management, HSE**

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# Standard approach: Building The Curve

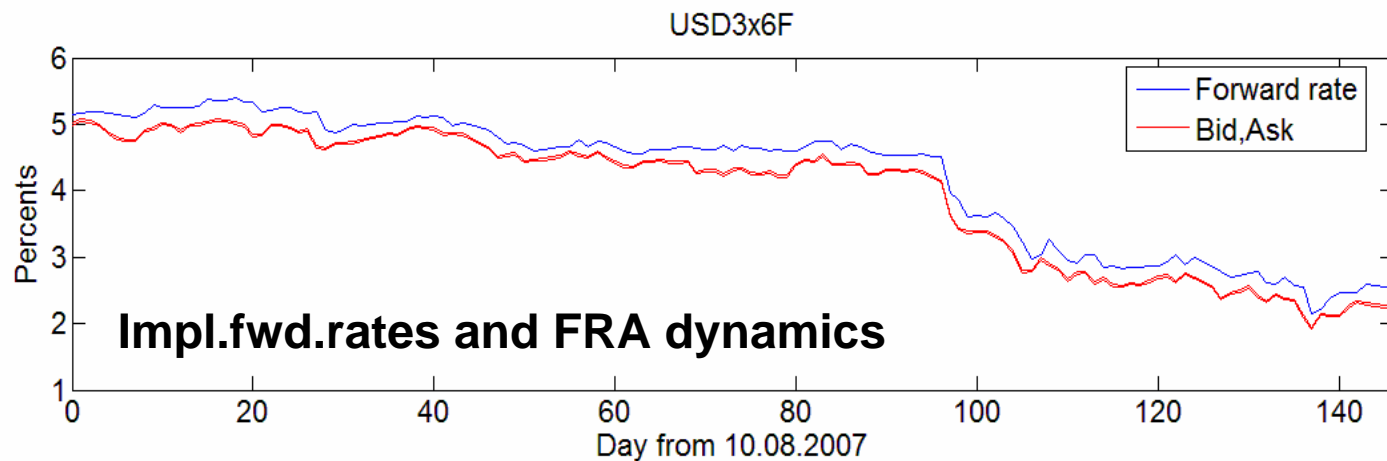
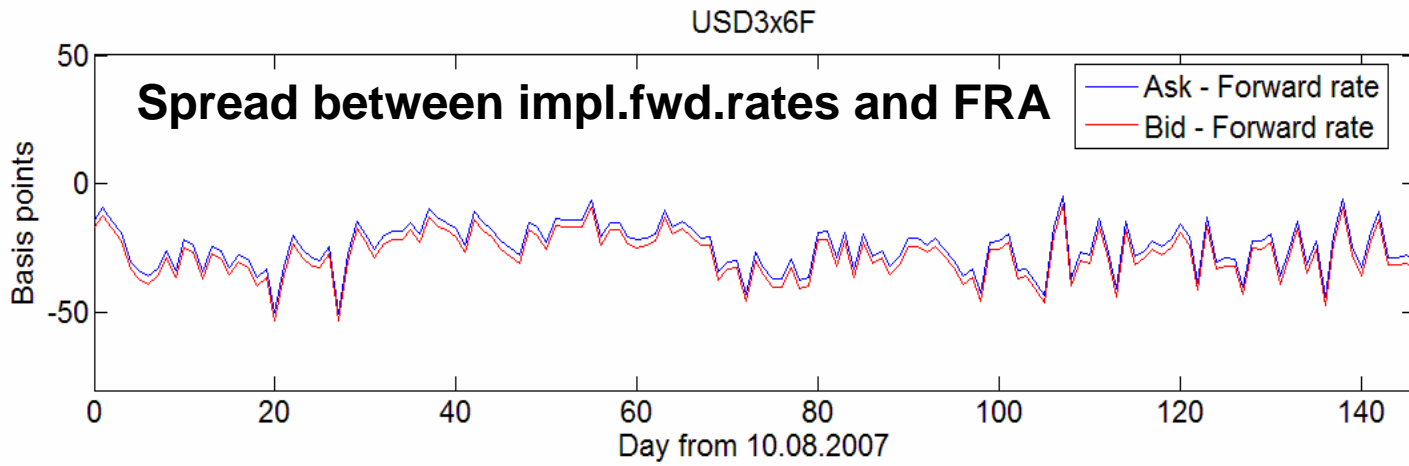
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- Curve coupling:
  - basis rates for short-term, < 1 Y (Libor, Euribor, MosPrime rates)
  - Swap rates for long term
- Expectations: The Curve should
  - Reflect interbank borrowing / lending business
  - Price the derivatives
  - Be simple



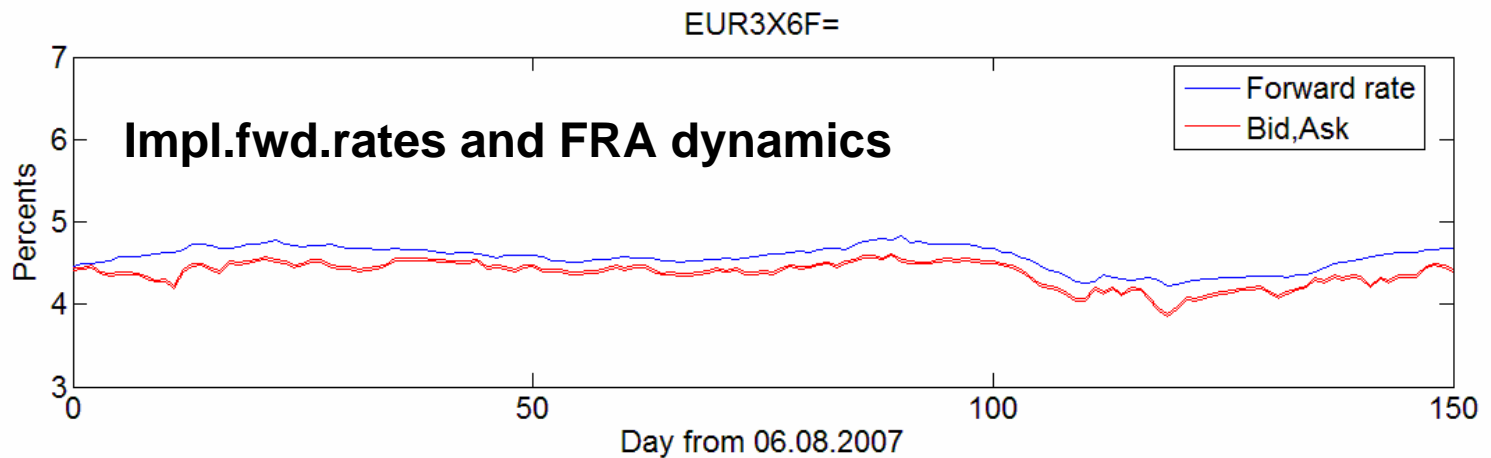
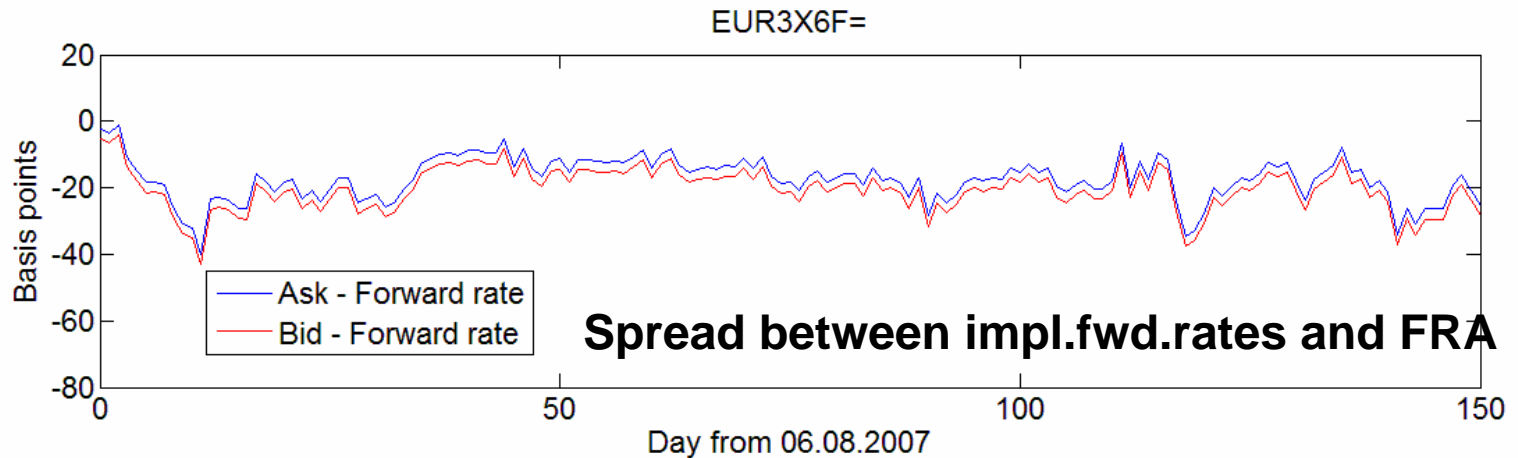
# Evidences of the standard approach's artifacts since the beginning of financial turmoil

- Implied forward rates versus USD FRA.
- Mispricing up to 50bp



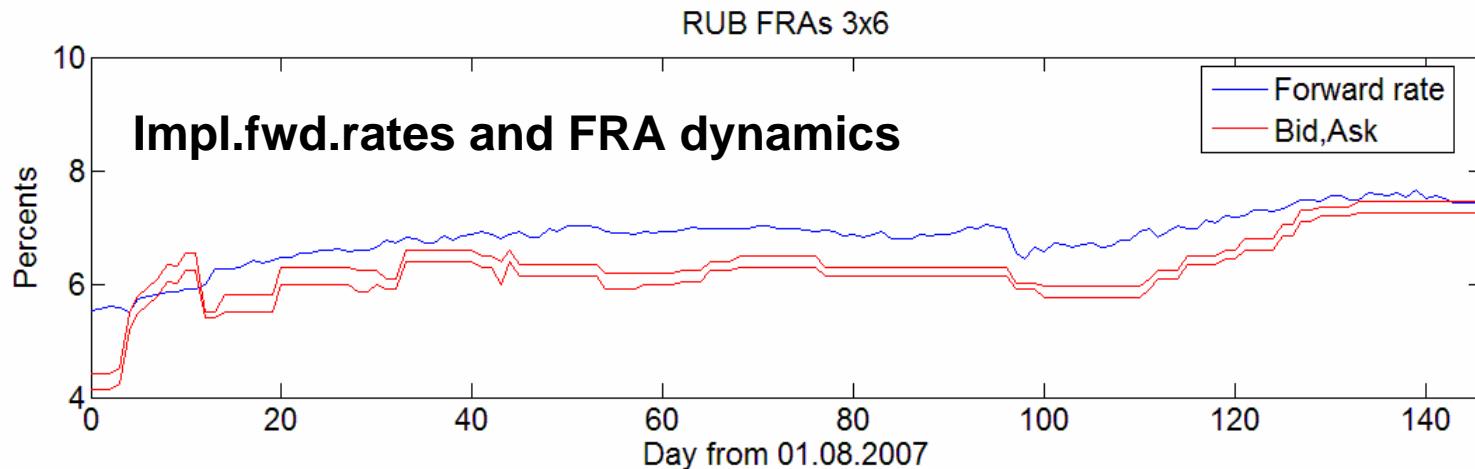
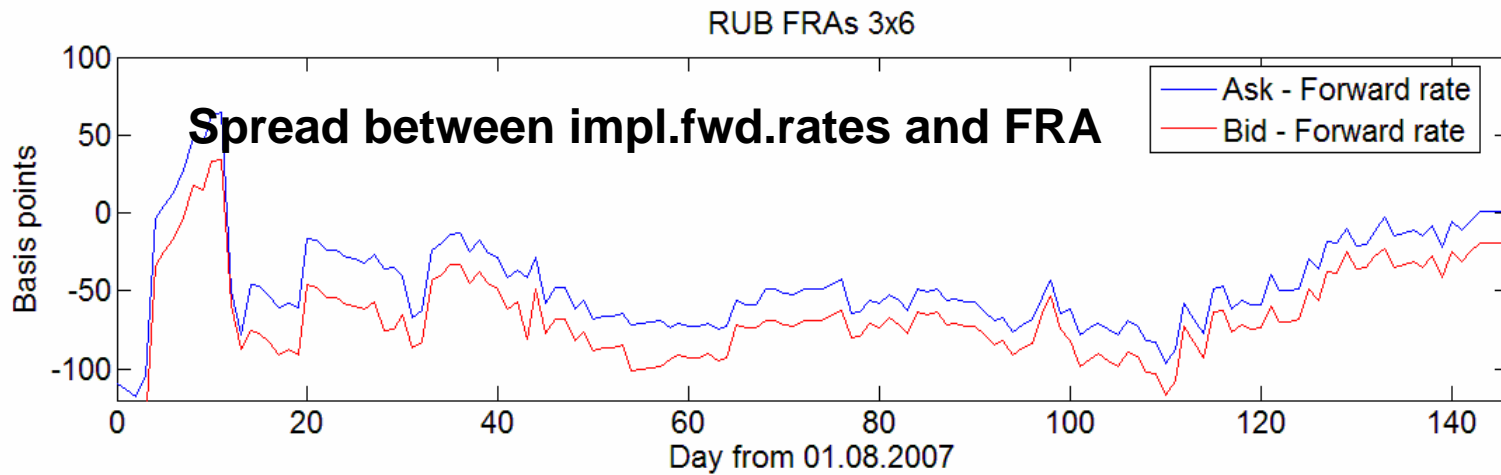
# Evidences of the standard approach's artifacts since the beginning of financial turmoil

- Implied forward rates versus EUR FRA.
- Mispricing up to 60bp



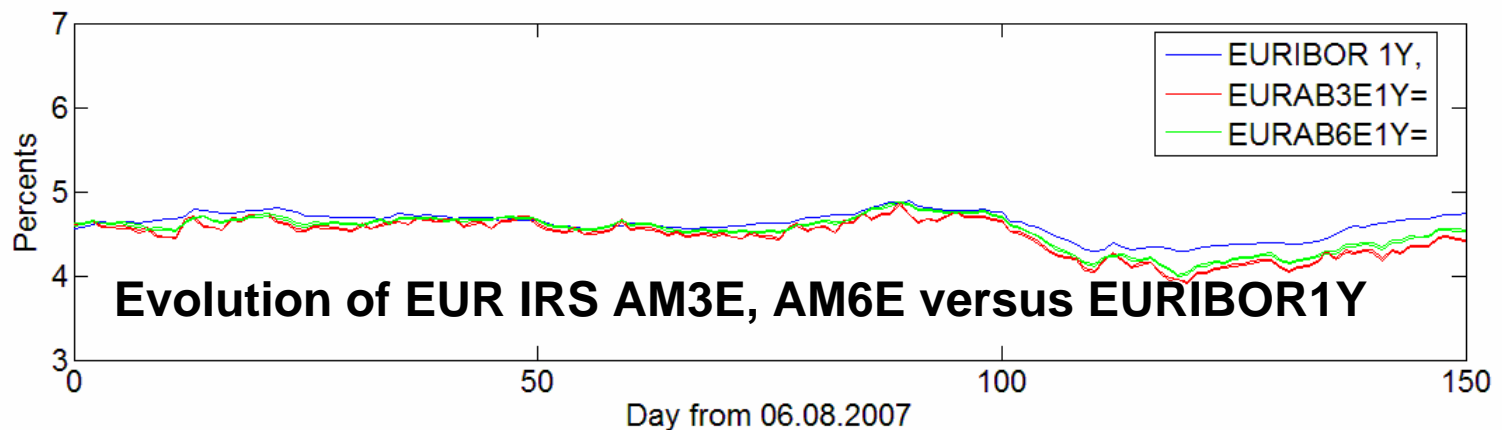
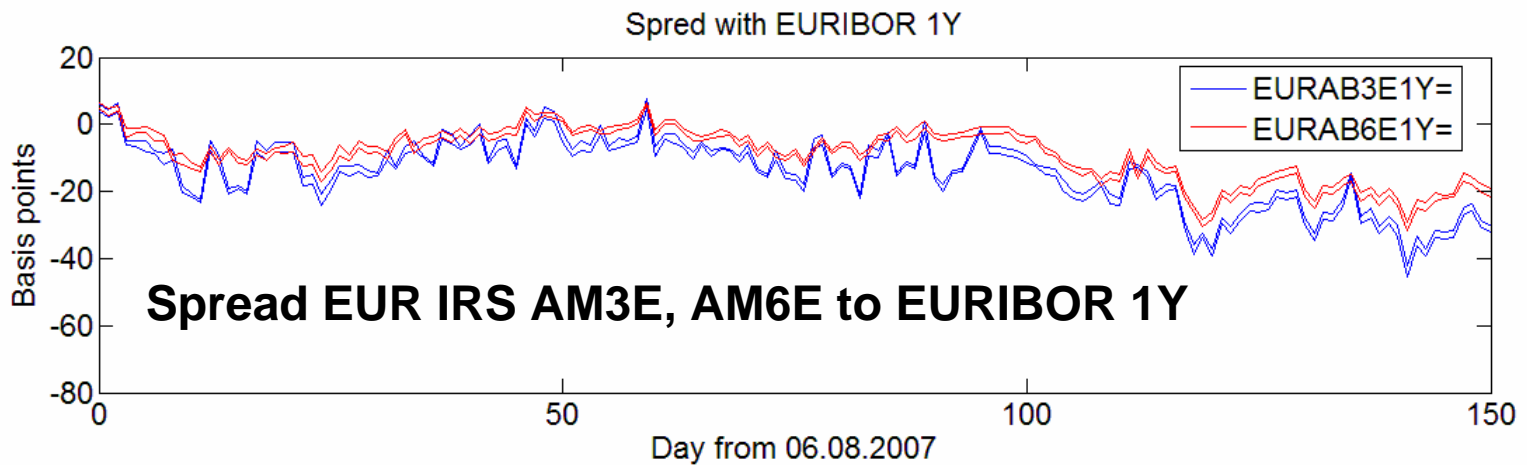
# Evidences of the standard approach's artifacts since the beginning of financial turmoil

- Implied forward rates versus RUB FRA.
- Mispricing up to 100 bp



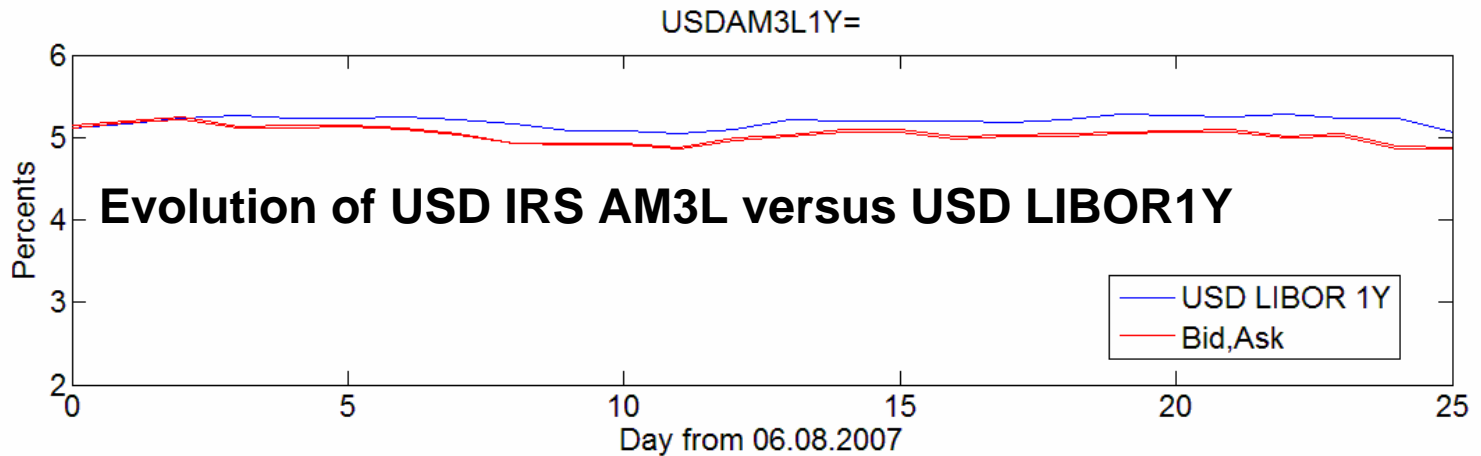
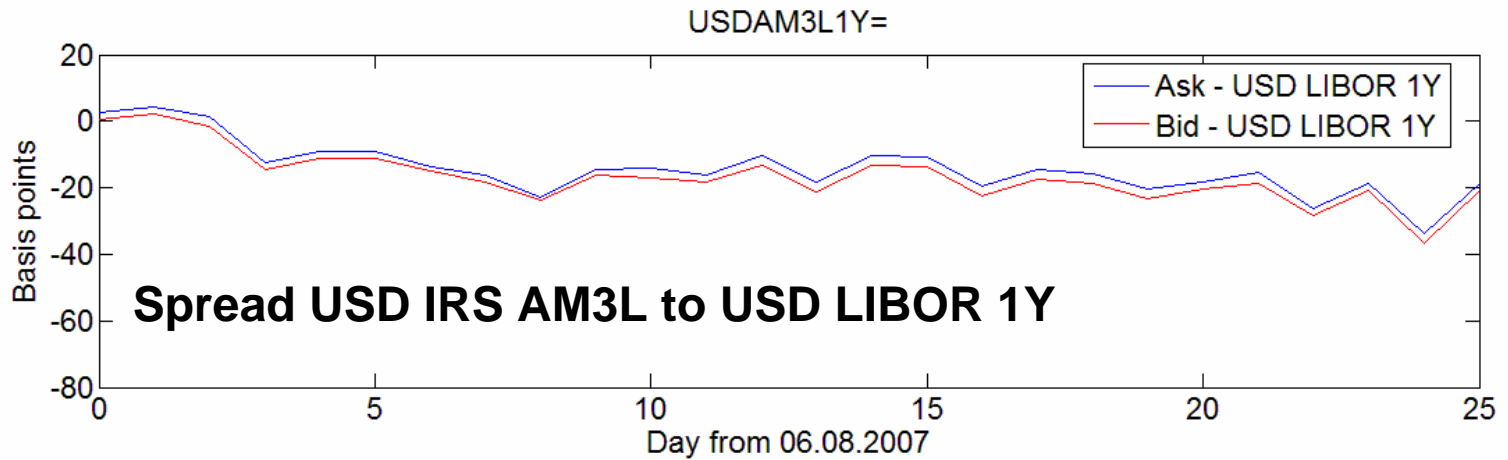
# Evidences of the standard approach's artifacts since the beginning of financial turmoil

- EURIBOR 1Y versus EUR IRS 1Y (AM3E, AM6E) rates
- Mispricing up to 40 bp



# Evidences of the standard approach's artifacts since the beginning of financial turmoil

- USD LIBOR 1Y versus USD IRS 1Y (AM3L)
- Mispricing up to 40 bp



# Arbitrage?

- There are no real arbitrage opportunities:
  - Basis rates are indicative for most of the market participants and does not reflect access to the market
- Moreover
  - Basis rates are indicatives for short- and middle-term banking liquidity management instruments and include liquidity cost
  - The Curve is used also for derivative pricing where no liquidity cost is included





# Diagnosis

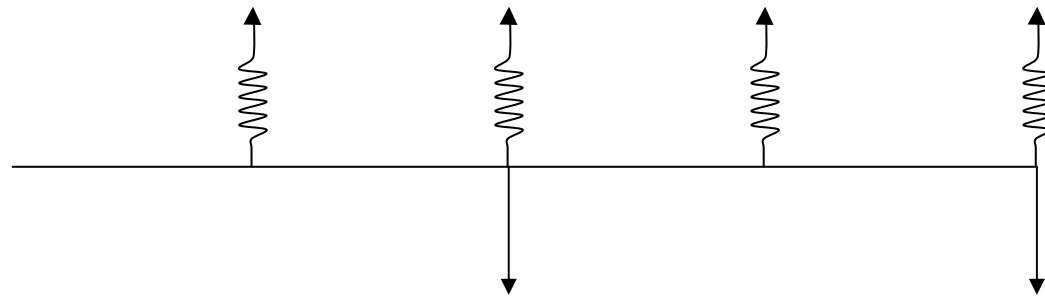
- For complex positions deviation from “fair value” of the portfolio could be very significant
- Opportunity for P/L manipulating is inadmissible from risk management point of view
- Standard approach can not solve all the problems at the same time.



- Target – model for pricing and revaluation of “plain vanilla” interest rate derivatives: IRS (fixed vs floating), FRA
- Analysis of model assumption
  - NPV based approach
  - $NPV = NPV(\text{fixed leg}) + NPV(\text{floating leg})$
  - Standard assumption:  $NPV(\text{floating leg})=1$  : possibility to refinance under floating rate

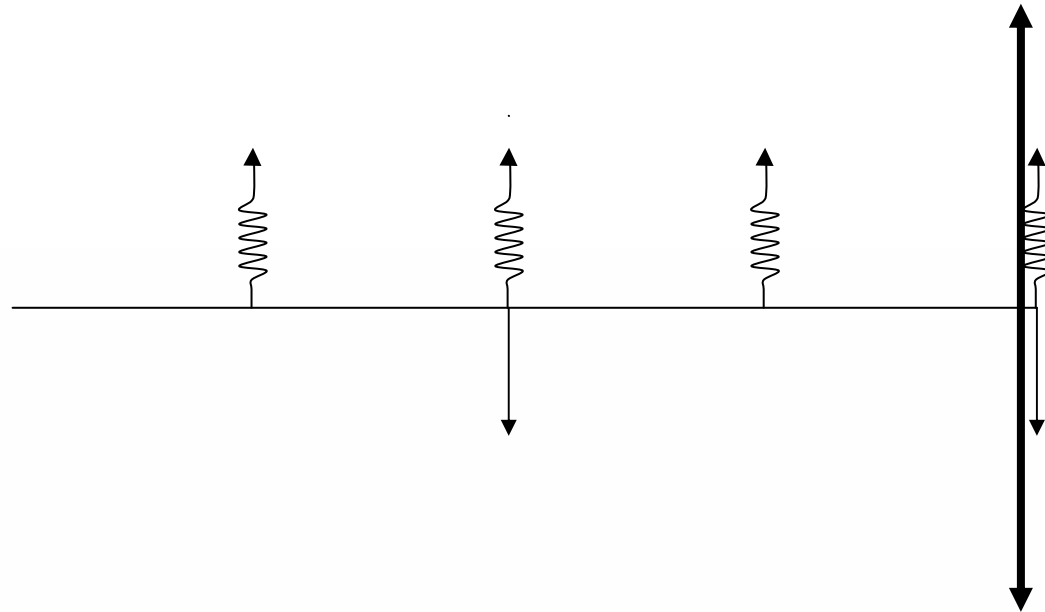
# Model assumption analysis: refinancing principle

- Consider cash flow of the 2 Y swap 6M floating versus annual fixed



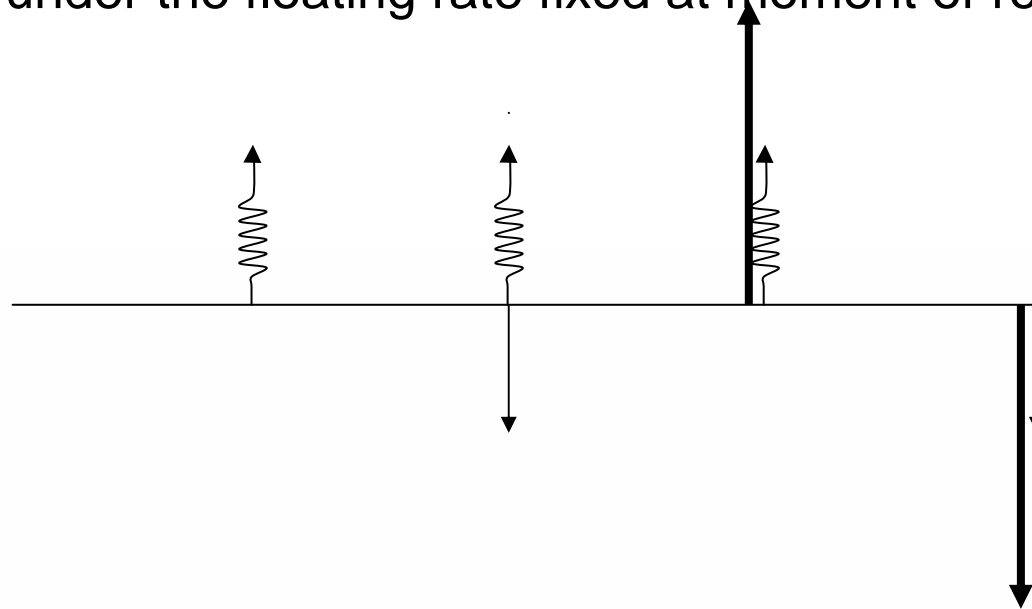
# Model assumption analysis: refinancing principle

- Consider cash flow of the 2 Y swap 6M floating versus annual fixed
- Plus / Minus face amount at maturity date



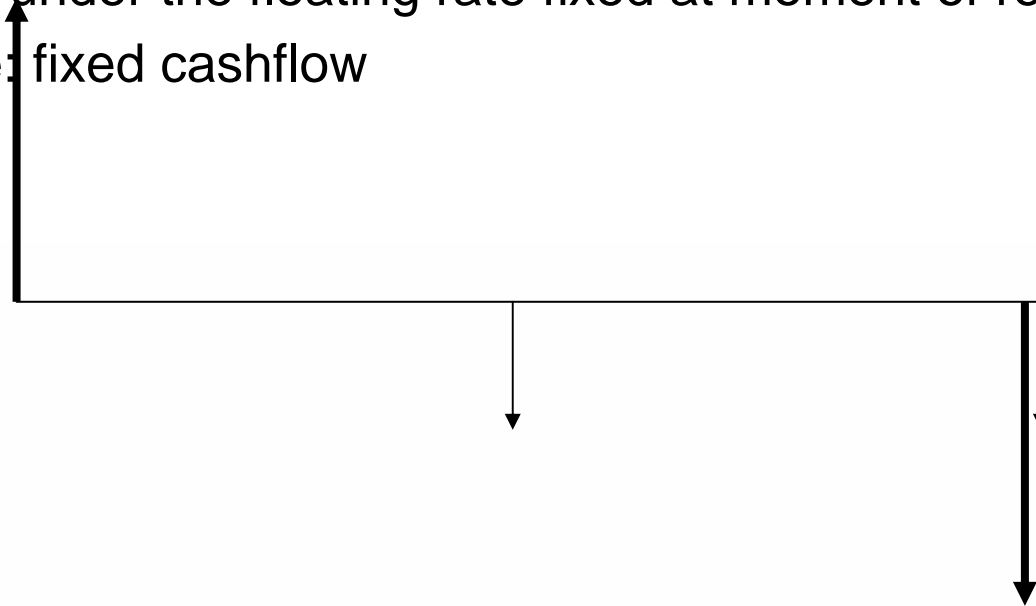
# Model assumption analysis: refinancing principle

- Consider cash flow of the 2 Y swap 6M floating versus annual fixed
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- Refinancing under the floating rate fixed at moment of refinancing



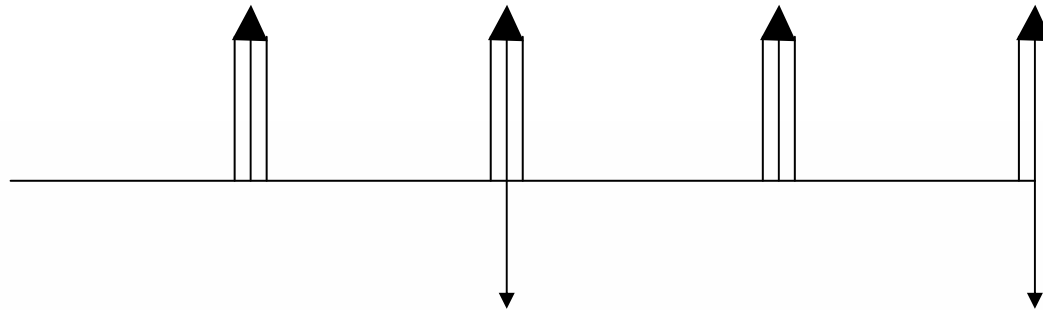
# Model assumption analysis: refinancing principle

- Consider cash flow of the 2 Y swap 6M floating versus annual fixed
- Plus / Minus face amount at maturity date
- Refinancing under the floating rate fixed at moment of refinancing
- Final picture: fixed cashflow



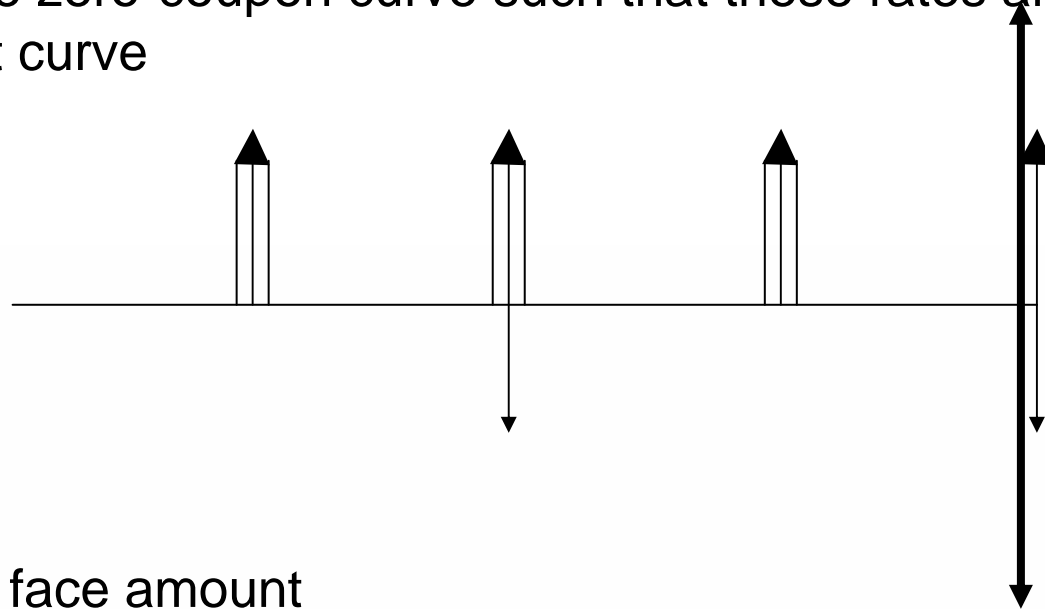
## Model assumption analysis: another view to refinancing principle

- Consider cash flow of the 2 Y swap 6M floating versus annual fixed
- Substitute floating cash flows with “expected” rates
- Lets build the zero-coupon curve such that these rates are forward rates for that curve



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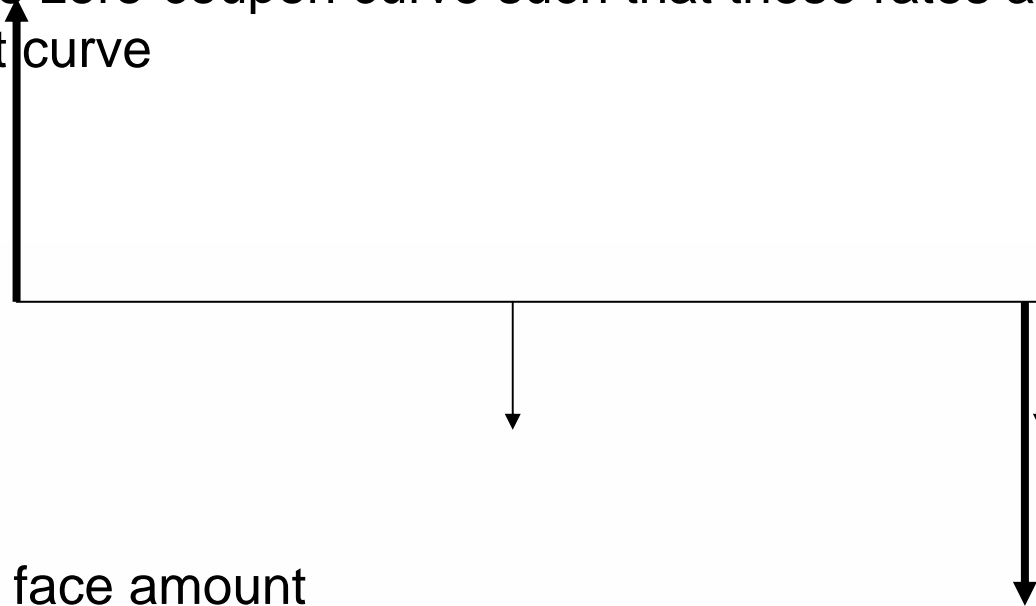
- Plus / Minus face amount





## Model assumption analysis: another view to refinancing principle

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- Plus / Minus face amount
- Strict equivalence from NPV point of view to the same cash flow without refinancing assumption



# Problem statement. 1

- The approach is admissible when we use the one curve for group of instruments on the same floating rate.
- Libor3M
  - IRS Libor3M vs. fixed (annual money) 1Y, 2Y, ...
  - FRA 1x4M, 2x5M, ...
- Euribor3M
  - IRS Euribor3M vs AM 1Y, 2Y, ...
  - FRA 1x4M, 2x5M, ...
- Euribor6M
  - IRS Euribor6M vs AM 1Y, 2Y, ...
  - FRA 1x7M, 2x8M, 3x9M, ...
- MosPrime3M
  - IRS MosPrime3M vs AM 1Y, 2Y, ...
  - FRA 1x4M, 2x5M, ...



## Problem statement. 2

- Discount function

$$d(t) = e^{-r(t)t} = e^{-\int_0^t f(s)ds}$$

- Monotony conditions

$$d_i = d(t_i), \quad d_s < d_t, s > t; s, t = 0, \dots, n, \quad d > 0, d < 1.$$

$$P_k = \sum_{i=0}^n C_{i,k} d(t_i)$$

- The system is underdetermined.
- Discount function values are desired in intermediate points
- Smoothness condition



# Problem statement. 3

- Adams and Van Deventer (1994), Smirnov and Zacharov (2003);
- Seek solution in the form

$$d = (d(t_0), \dots, d(t_n)) = (d_0, \dots, d_n)$$

$$t_i, i = 0, \dots, n,$$

- Tikhonov regularization

$$\sum_{i=1}^N \alpha_i \left( \sum_{j=0}^n C_{ij} d_j \right)^2 + \varepsilon \sum_{j=2}^{n-1} \frac{(d_{j-1} - 2d_j + d_{j+1})^2}{(t_j - t_{j-1})^2} \longrightarrow \min$$



- Bounds for zero coupon curve values that satisfy:
  - Monotony of discount coefficients = non-negative forward rates
  - Implied rates should be within bid/offer spreads
  - Smirnov and Zacharov (2003)

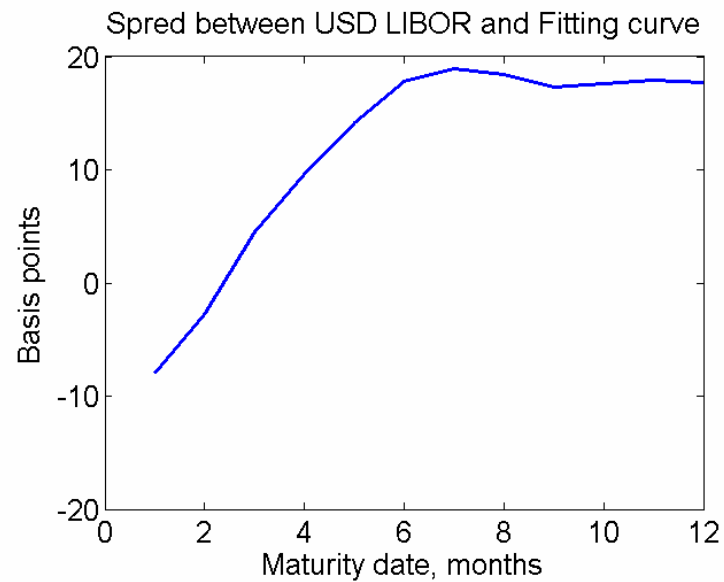
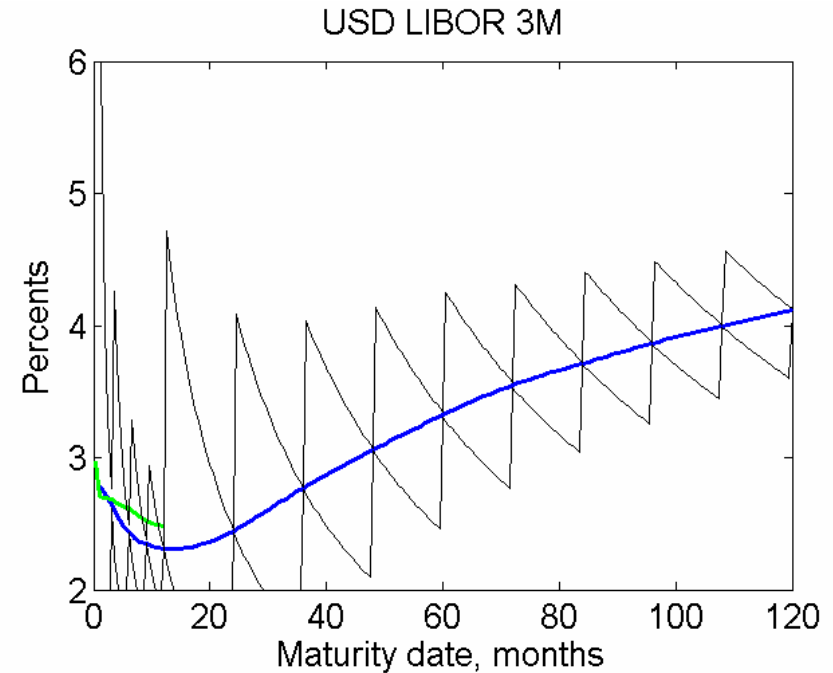
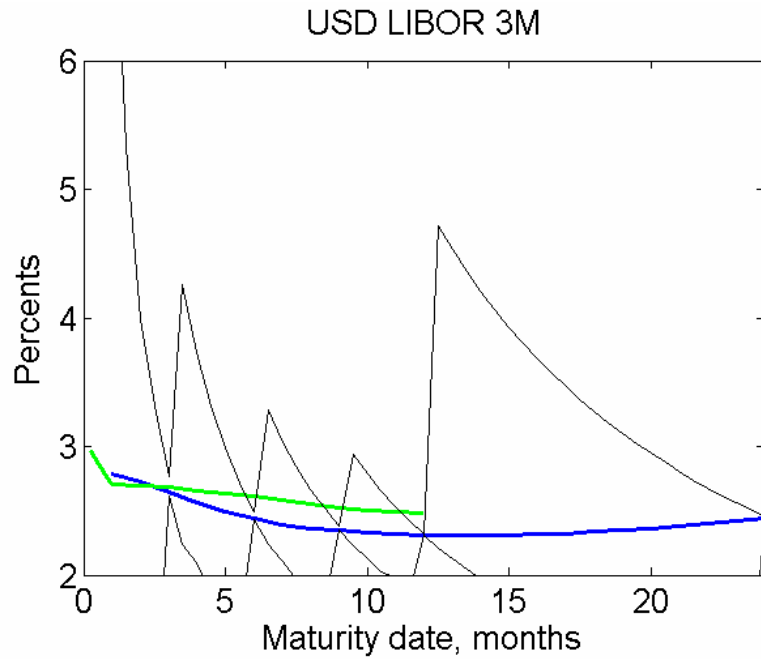


## Numerical example. Data.

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- Interest Rate Swap and Forward Rate Agreement bid-ask quotes on the time interval 01.08.2007 to 28.03.2008,  
Source: Reuters, GFI broker for RUB instruments
- All instruments are segregated into four group by underlying basis rate:  
USD LIBOR 3M, MOSPRIME 3M, EURIBOR 3M, EURIBOR 6M.

# Numerical example. USD Libor 3M

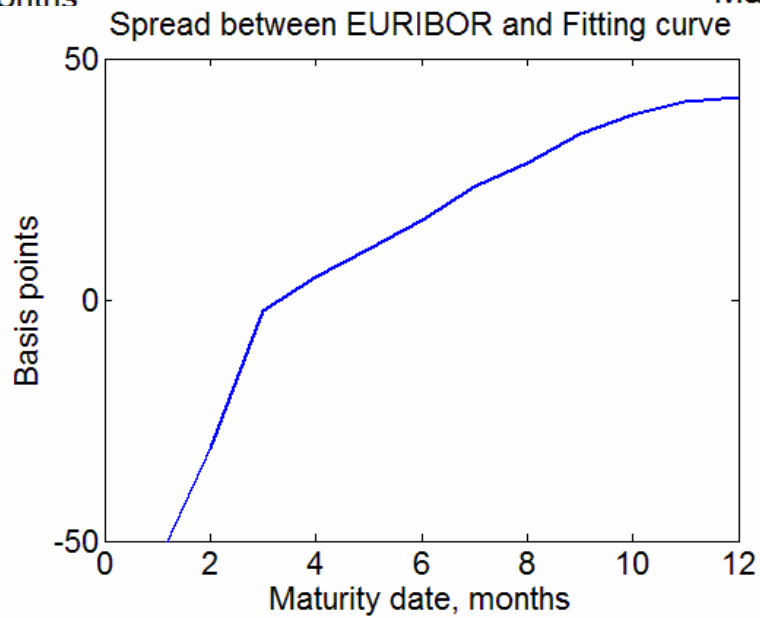
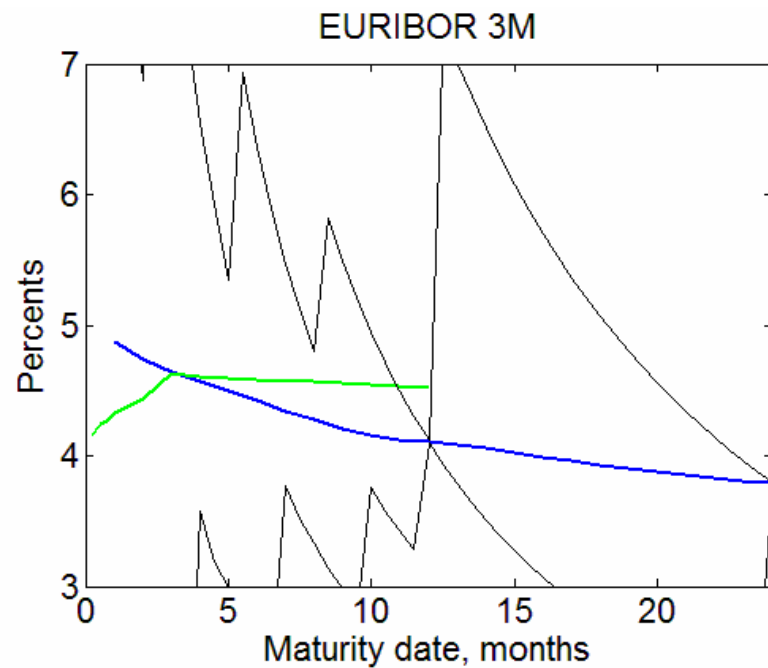
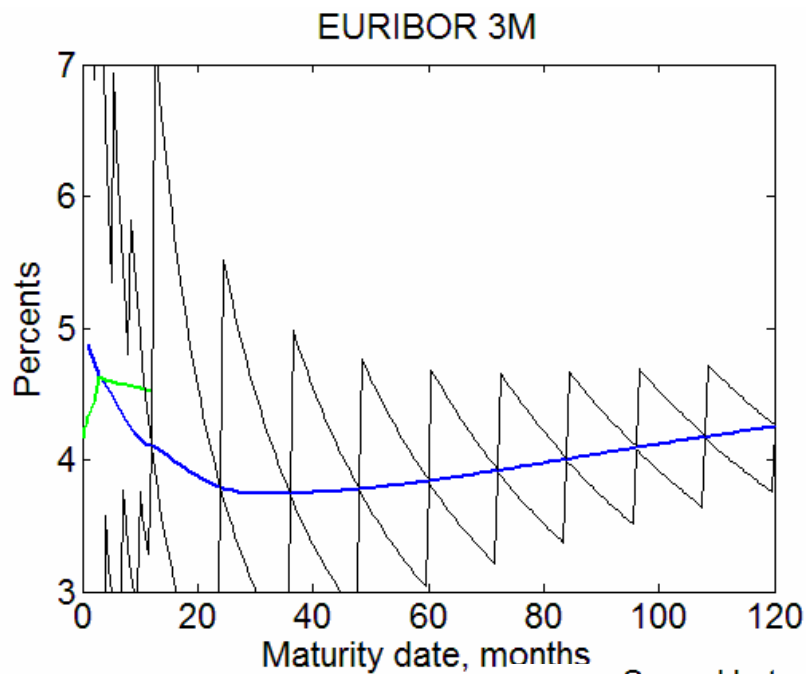


Liquidity cost relatively to 3-month liquidity

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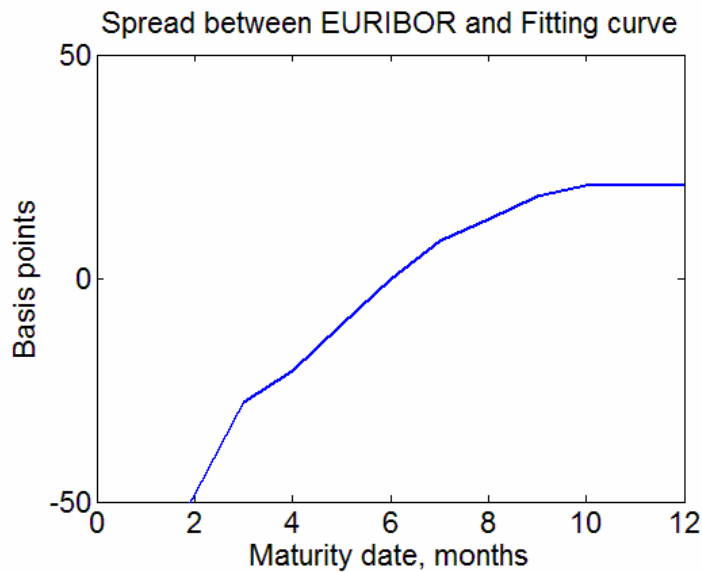
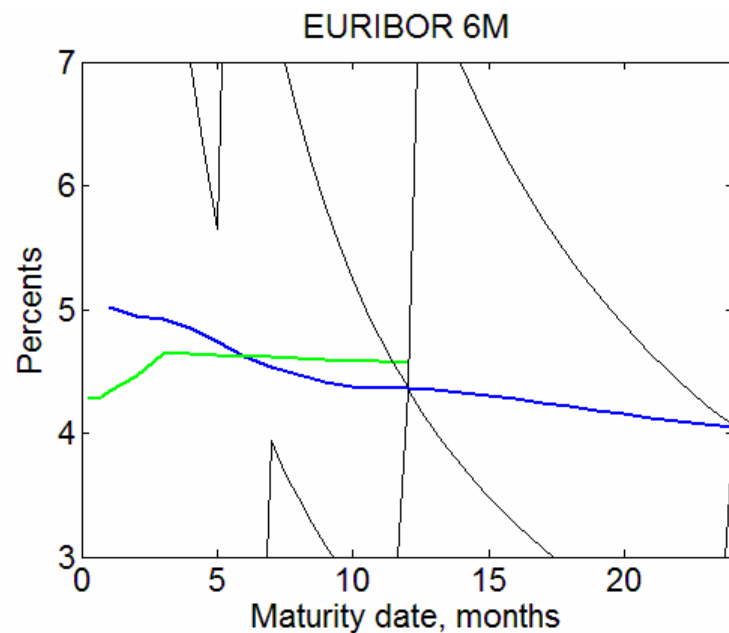
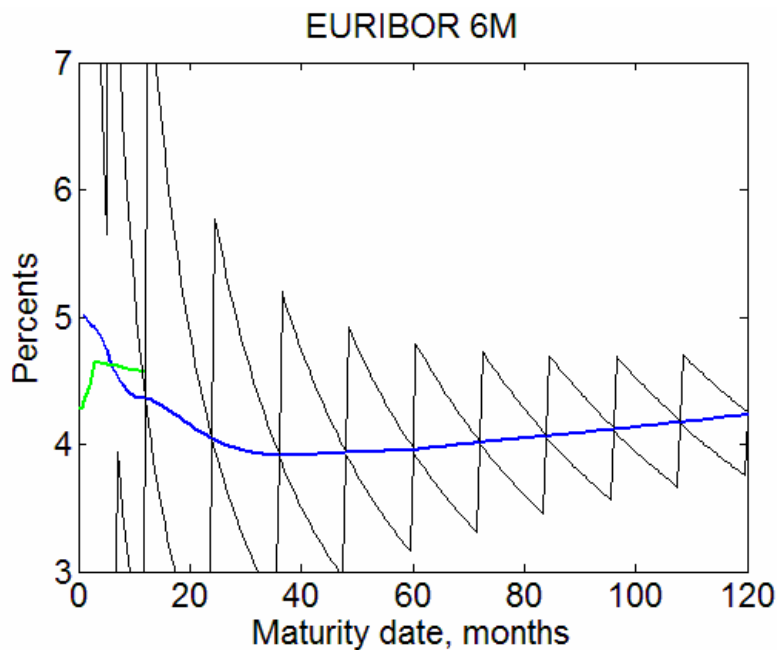


# Numerical example. EURIBOR 3M

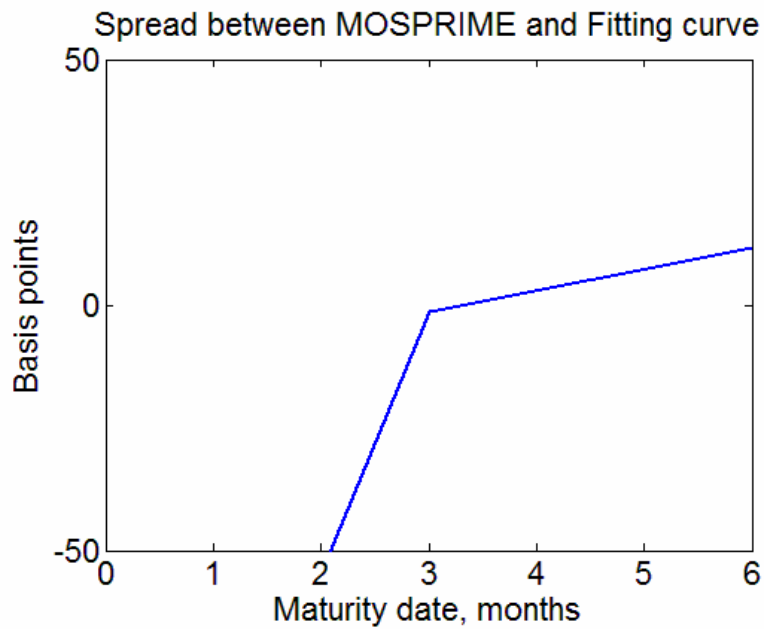
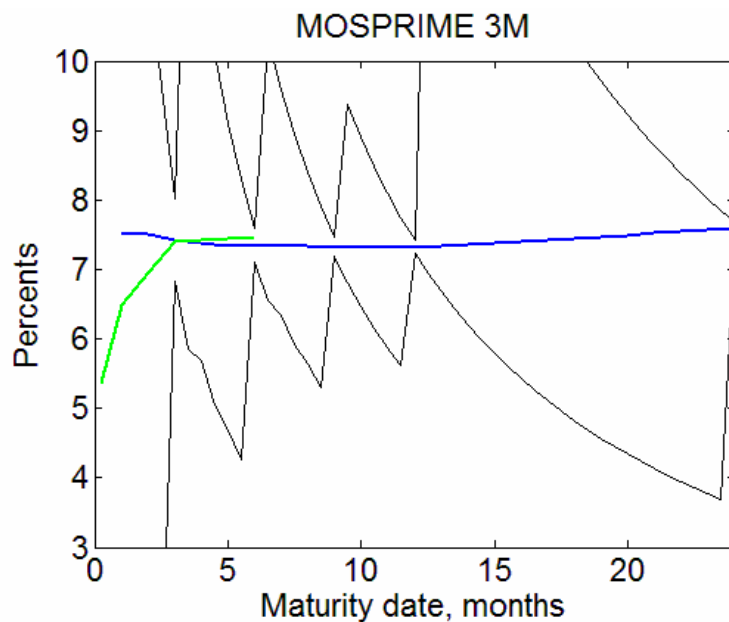
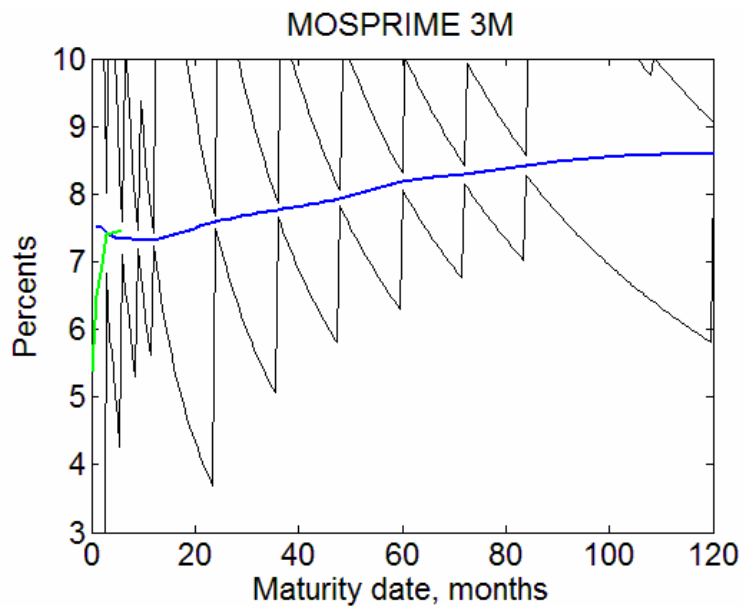




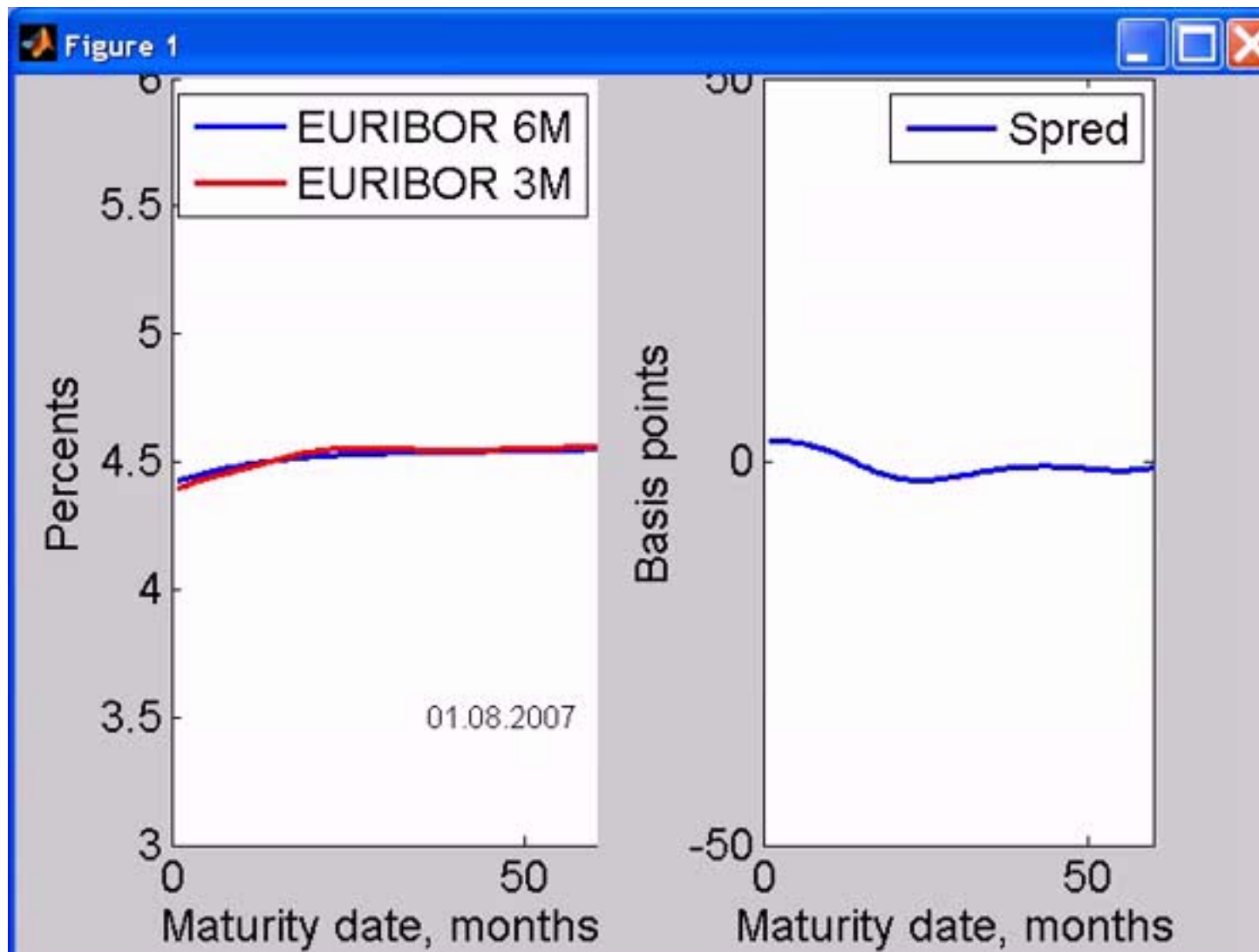
# Numerical example. EURIBOR 6M



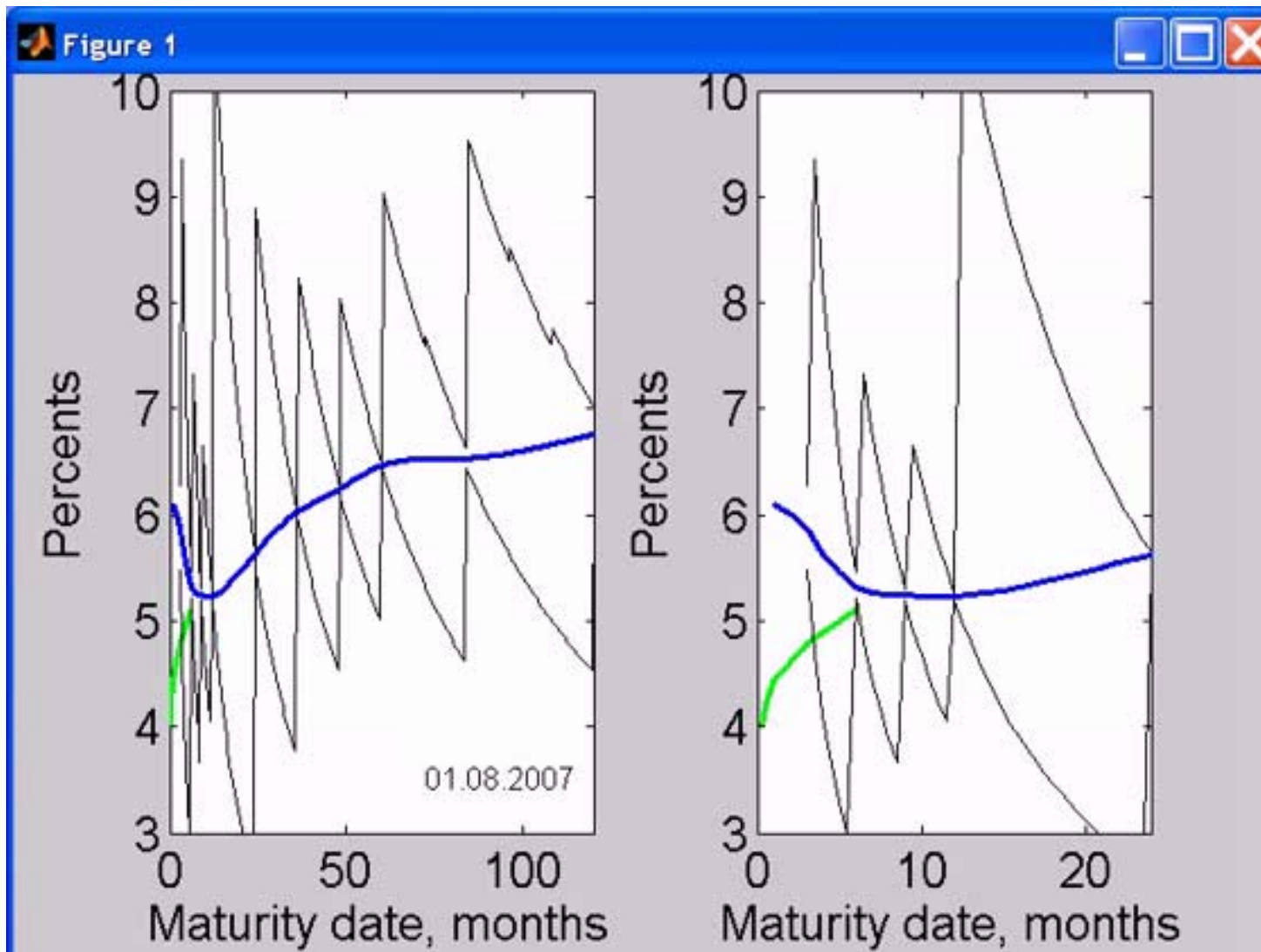
# Numerical example. MosPrime 3M



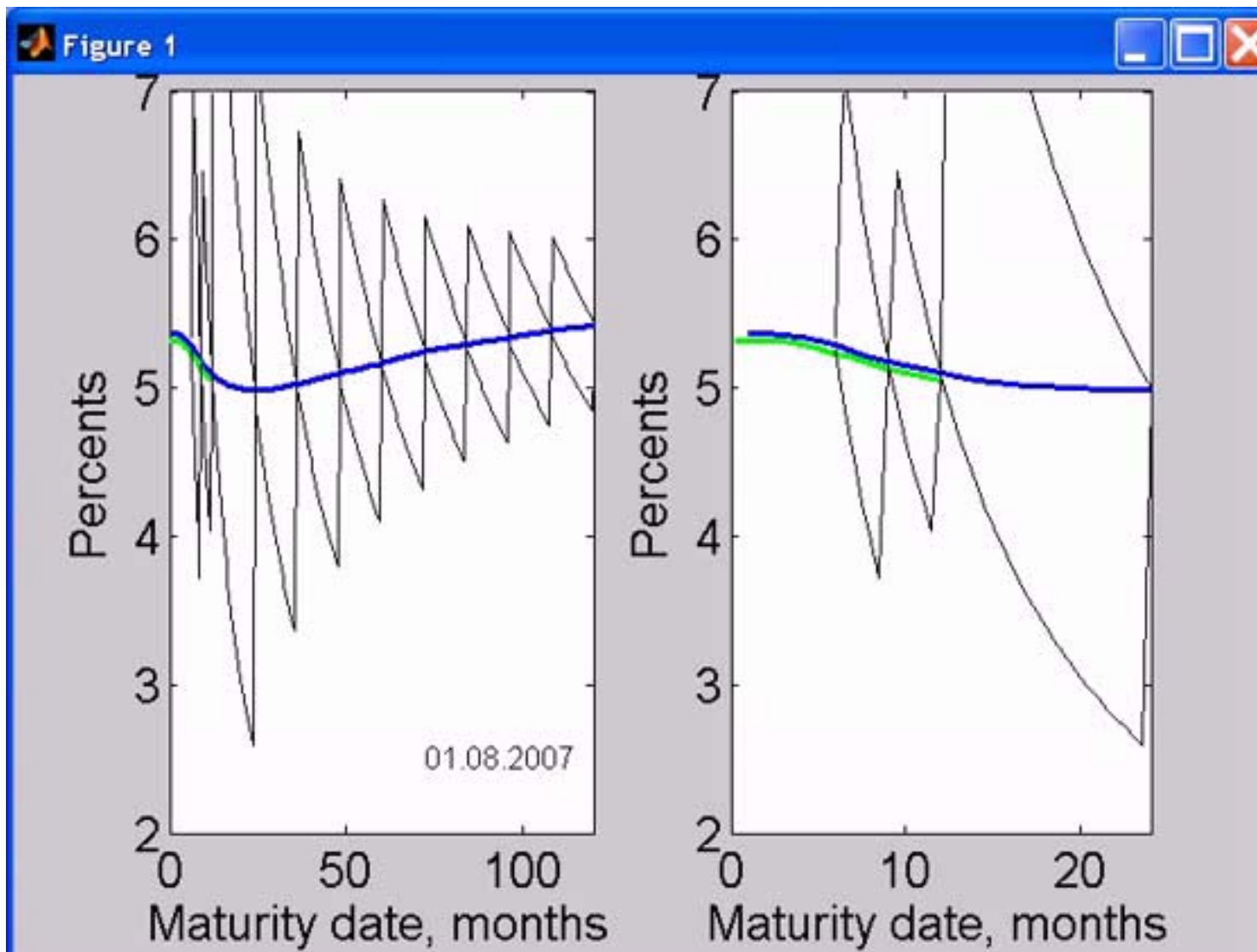
# Numerical example. Euribor 3M, 6M



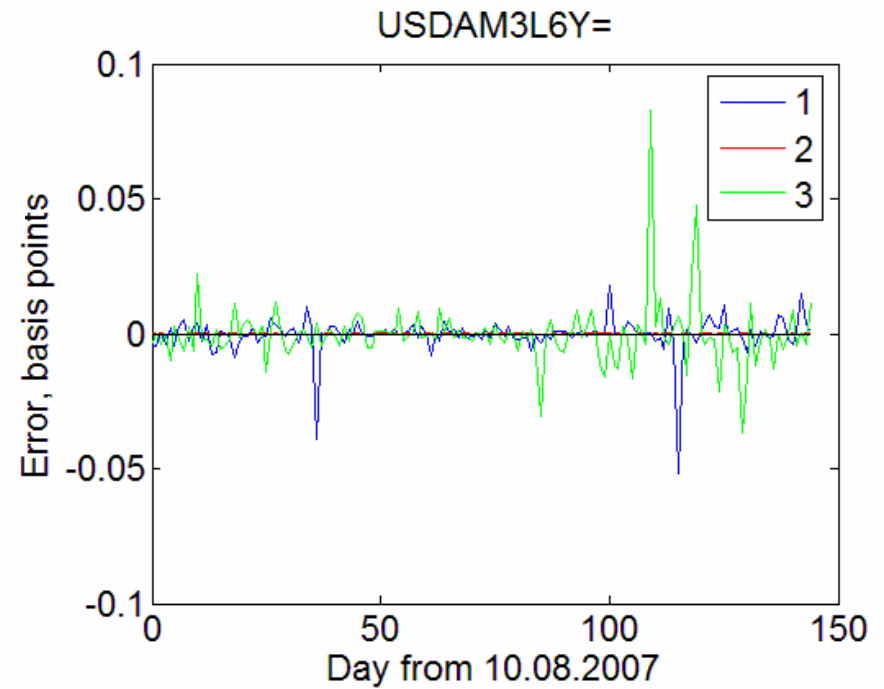
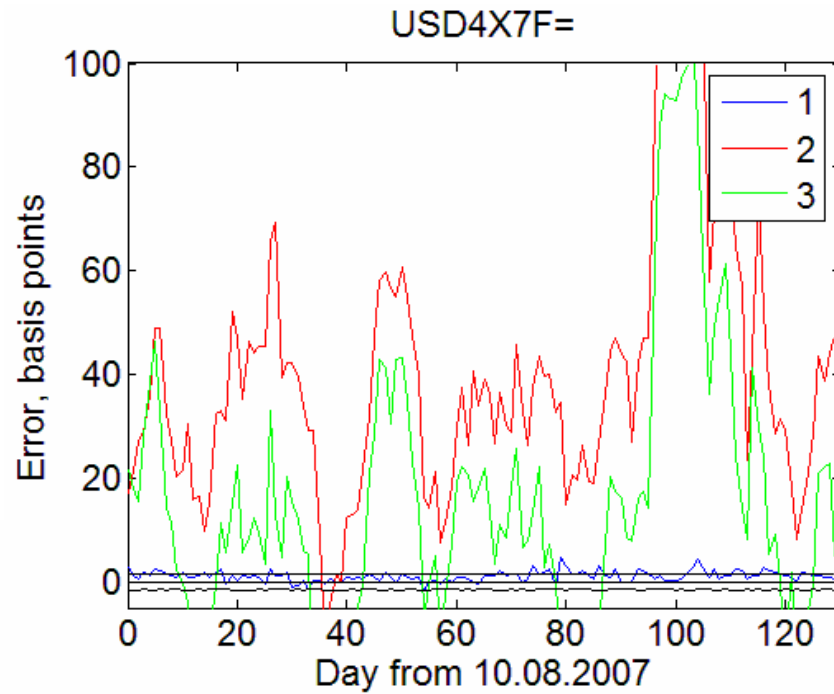
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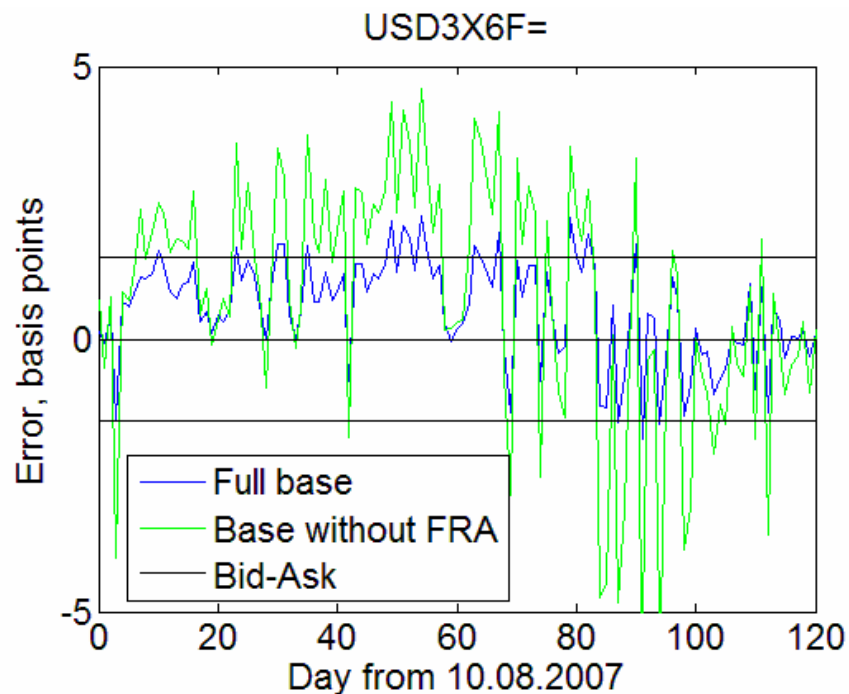
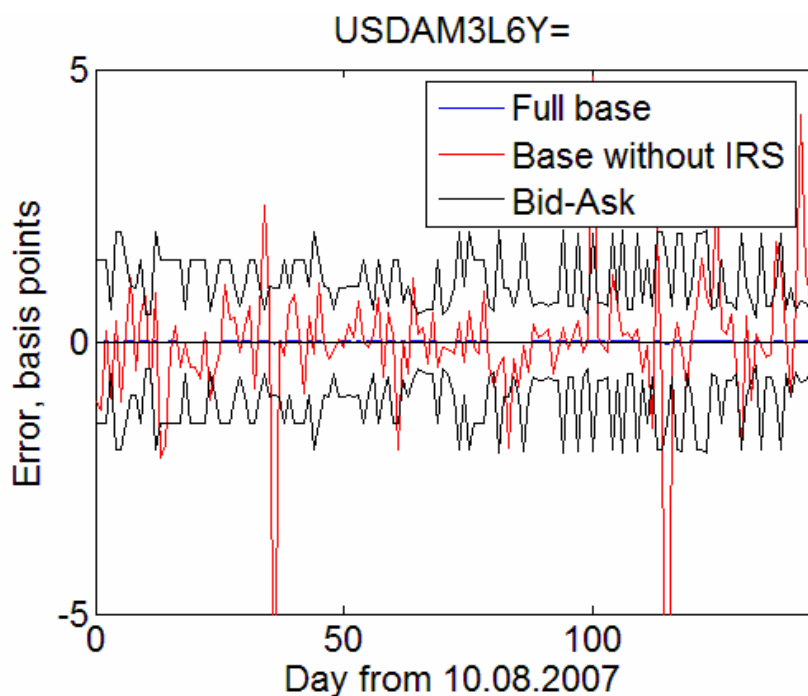
# Numerical example. USD Libor 3M



# Numerical example. Dynamics of error



# Stability of the allocation base



# Thank you!

Any questions?

Ilya Polimatidi

[ipolimatidi@gmail.com](mailto:ipolimatidi@gmail.com)

[ilya.polimatidi@unicreditgroup.ru](mailto:ilya.polimatidi@unicreditgroup.ru)

Olga Karnauhova

[olga.karnauhova@gmail.com](mailto:olga.karnauhova@gmail.com)

